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**CMSC 203 – Discrete Structures**
**Section 0401, Spring 2005**
**Example Set 1**

1.1.7. What are the contrapositive, the converse,a nd the inverse of the implication

"The home team wins whenever it is raining."?

*Solution*: First, it is best to rewrite the implication for clarity.

If it is raining, then the home team wins.

**Converse**: If the home team wins, then it is raining.

**Contrapositive**: If the home team does not win, then it is not raining.

**Inverse**: If it is not raining, then the home team does not win.

1.1.10. How can this English sentence be translated into a logical expression?

"You cannot ride the roller coaster if you are under four feet tall unless you are older than 16 years old."

*Solution*: The first thing that must be done is to use propositional variables to represent each of the sentence parts. Let

- $q$ be "You cannot ride the roller coaster,"
- $r$ be "You are under four feet tall," and
- $s$ be "You are older than 16 years old."

Then the sentence can be translated into

$$(r \wedge \neg s) \rightarrow \neg q.$$

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1.1.12. Determine whether the following system specifications are consistent:

> "The diagnostic message is stored in the buffer or it is retransmitted."
> "The diagnostic message is not stored in the buffer."
> "If the diagnostic message is stored in the buffer, then it is retranmitted."

*Solution*: First, translate the sentence parts into propositional variables:

- $p$ is "The diagnostic message is stored in the buffer" and
- $q$ is "The diagnostic message is retransmitted."

Then the specification can be rewritten as

$$p \vee q,$$

$$\neg p,$$

$$p \rightarrow q.$$

Now, for the system specification to be consistent, we must be able to assign truth values to the variables to make all specifications true. So, $p$ must be false to make $\neg p$ true. Since $p \vee q$ must be true, but $p$ is false, then $q$ must be true. Then $p \rightarrow q$ is also true as $p$ is false and $q$ is true. Therefore, the system specification is consistent.

1.2.6. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

*Solution*: It is necessary to use logical equivalences to show that the statement above is a tautology.

$$\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{(by implication)} \\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{(by De Morgan)} \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{(by associative and commutative)} \\
&\equiv \mathbf{T} \vee \mathbf{T} && \text{(by negation)} \\
&\equiv \mathbf{T}
\end{aligned}$$