Table of Fourier Transform Pairs

Function, f(t)	Fourier Transform, F(ω)
Definition of Inverse Fourier Transform $$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\, d\omega$$	*Definition of Fourier Transform* $$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\, dt$$
$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
$f(\alpha t)$	$\dfrac{1}{\lvert\alpha\rvert} F(\dfrac{\omega}{\alpha})$
$F(t)$	$2\pi f(-\omega)$
$\dfrac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$(-jt)^n f(t)$	$\dfrac{d^n F(\omega)}{d\omega^n}$
$\displaystyle\int_{-\infty}^{t} f(\tau)\, d\tau$	$\dfrac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
sgn (t)	$\dfrac{2}{j\omega}$

Fourier Transform Table

$F(t)$	$\widehat{F}(\omega)$	Notes	
$f(t)$	$\displaystyle\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	Definition.	(1)
$\displaystyle\frac{1}{2\pi}\int_{-\infty}^{\infty}\widehat{f}(\omega)e^{i\omega t}\,d\omega$	$\widehat{f}(\omega)$	Inversion formula.	(2)
$\widehat{f}(-t)$	$2\pi f(\omega)$	Duality property.	(3)
$e^{-at}u(t)$	$\dfrac{1}{a+i\omega}$	a constant, $\Re e(a) > 0$	(4)
$e^{-a|t|}$	$\dfrac{2a}{a^2+\omega^2}$	a constant, $\Re e(a) > 0$	(5)
$\beta(t) = \begin{cases} 1, & \text{if } |t| < 1, \\ 0, & \text{if } |t| > 1 \end{cases}$	$2\operatorname{sinc}(\omega) = 2\dfrac{\sin(\omega)}{\omega}$	Boxcar in time.	(6)
$\dfrac{1}{\pi}\operatorname{sinc}(t)$	$\beta(\omega)$	Boxcar in frequency.	(7)
$f'(t)$	$i\omega\widehat{f}(\omega)$	Derivative in time.	(8)
$f''(t)$	$(i\omega)^2\widehat{f}(\omega)$	Higher derivatives similar.	(9)
$tf(t)$	$i\dfrac{d}{d\omega}\widehat{f}(\omega)$	Derivative in frequency.	(10)
$t^2 f(t)$	$i^2\dfrac{d^2}{d\omega^2}\widehat{f}(\omega)$	Higher derivatives similar.	(11)
$e^{i\omega_0 t}f(t)$	$\widehat{f}(\omega-\omega_0)$	Modulation property.	(12)
$f\left(\dfrac{t-t_0}{k}\right)$	$ke^{-i\omega t_0}\widehat{f}(k\omega)$	Time shift and squeeze.	(13)
$(f*g)(t)$	$\widehat{f}(\omega)\widehat{g}(\omega)$	Convolution in time.	(14)
$u(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t > 0 \end{cases}$	$\dfrac{1}{i\omega} + \pi\delta(\omega)$	Heaviside step function.	(15)
$\delta(t-t_0)f(t)$	$e^{-i\omega t_0}f(t_0)$	Assumes f continuous at t_0.	(16)
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	Useful for $\sin(\omega_0 t)$, $\cos(\omega_0 t)$.	(17)

Convolution:
$$(f*g)(t) = \int_{-\infty}^{\infty} f(t-u)g(u)\,du = \int_{-\infty}^{\infty} f(u)g(t-u)\,du.$$

Parseval:
$$\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\widehat{f}(\omega)\right|^2 d\omega.$$

$j\dfrac{1}{\pi t}$	$\operatorname{sgn}(\omega)$
$u(t)$	$\pi\delta(\omega) + \dfrac{1}{j\omega}$
$\displaystyle\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$	$2\pi \displaystyle\sum_{n=-\infty}^{\infty} F_n \delta(\omega - n\omega_0)$
$rect(\dfrac{t}{\tau})$	$\tau Sa(\dfrac{\omega\tau}{2})$
$\dfrac{B}{2\pi} Sa(\dfrac{Bt}{2})$	$rect(\dfrac{\omega}{B})$
$tri(t)$	$Sa^2(\dfrac{\omega}{2})$
$A\cos(\dfrac{\pi t}{2\tau})rect(\dfrac{t}{2\tau})$	$\dfrac{A\pi}{\tau}\dfrac{\cos(\omega\tau)}{(\pi/2\tau)^2 - \omega^2}$
$\cos(\omega_0 t)$	$\pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$
$\sin(\omega_0 t)$	$\dfrac{\pi}{j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$
$u(t)\cos(\omega_0 t)$	$\dfrac{\pi}{2}\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right] + \dfrac{j\omega}{\omega_0^2 - \omega^2}$
$u(t)\sin(\omega_0 t)$	$\dfrac{\pi}{2j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right] + \dfrac{\omega^2}{\omega_0^2 - \omega^2}$
$u(t)e^{-\alpha t}\cos(\omega_0 t)$	$\dfrac{(\alpha + j\omega)}{\omega_0^2 + (\alpha + j\omega)^2}$

$u(t)e^{-\alpha t}\sin(\omega_0 t)$	$\dfrac{\omega_0}{\omega_0^2 + (\alpha + j\omega)^2}$
$e^{-\alpha|t|}$	$\dfrac{2\alpha}{\alpha^2 + \omega^2}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$
$u(t)e^{-\alpha t}$	$\dfrac{1}{\alpha + j\omega}$
$u(t)te^{-\alpha t}$	$\dfrac{1}{(\alpha + j\omega)^2}$

➢ *Trigonometric Fourier Series*

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(\omega_0 n t) + b_n \sin(\omega_0 n t)\right)$$

where

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad , \quad a_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_0 n t)dt \quad ,\text{and}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_0 n t)dt$$

➢ *Complex Exponential Fourier Series*

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega n t} \quad ,\text{where} \quad F_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_0 n t}dt$$

Some Useful Mathematical Relationships

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$2\cos^2(x) = 1 + \cos(2x)$$

$$2\sin^2(x) = 1 - \cos(2x)$$

$$\cos^2(x) + \sin^2(x) = 1$$

$$2\cos(x)\cos(y) = \cos(x - y) + \cos(x + y)$$

$$2\sin(x)\sin(y) = \cos(x - y) - \cos(x + y)$$

$$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y)$$

Useful Integrals

$\int \cos(x)dx$	$\sin(x)$
$\int \sin(x)dx$	$-\cos(x)$
$\int x\cos(x)dx$	$\cos(x) + x\sin(x)$
$\int x\sin(x)dx$	$\sin(x) - x\cos(x)$
$\int x^2 \cos(x)dx$	$2x\cos(x) + (x^2 - 2)\sin(x)$
$\int x^2 \sin(x)dx$	$2x\sin(x) - (x^2 - 2)\cos(x)$
$\int e^{ax}dx$	$\dfrac{e^{ax}}{a}$
$\int xe^{ax}dx$	$e^{ax}\left[\dfrac{x}{a} - \dfrac{1}{a^2}\right]$
$\int x^2 e^{ax}dx$	$e^{ax}\left[\dfrac{x^2}{a} - \dfrac{2x}{a^2} - \dfrac{2}{a^3}\right]$
$\int \dfrac{dx}{\alpha + \beta x}$	$\dfrac{1}{\beta}\ln\lvert \alpha + \beta x\rvert$
$\int \dfrac{dx}{\alpha^2 + \beta^2 x^2}$	$\dfrac{1}{\alpha\beta}\tan^{-1}(\dfrac{\beta x}{\alpha})$

Name	Transform	Inverse transform
FT	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}$	$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega)e^{j\omega t} d\omega$
Fourier series	$\hat{f}_0[m] = \frac{1}{T} \int_0^T f_0(t)e^{-j2\pi mt/T} dt$	$f_0(t) = \sum_{m=-\infty}^{\infty} \hat{f}_0[m]e^{j2\pi mt/T}$
DTFT	$\hat{f}(\omega) = \sum_{n=-\infty}^{\infty} f[n]e^{-j\omega n}$	$f[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{f}(\omega)e^{j\omega n} d\omega$
DFT	$\hat{f}[m] = \sum_{n=0}^{N-1} f[n]e^{-j\frac{2\pi}{N}mn}$	$f[n] = \sum_{m=0}^{N-1} \hat{f}[m]e^{j\frac{2\pi}{N}mn}$