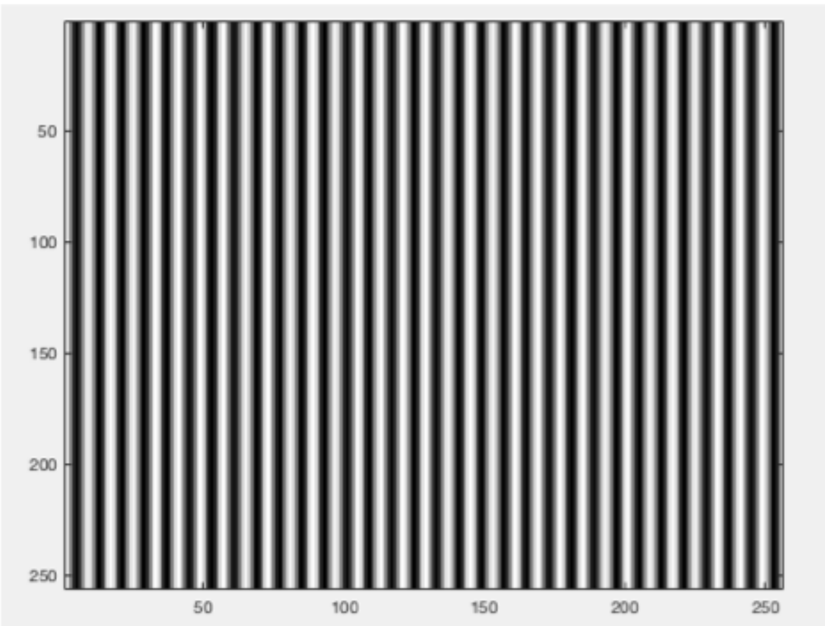
The 2D Fourier Transform

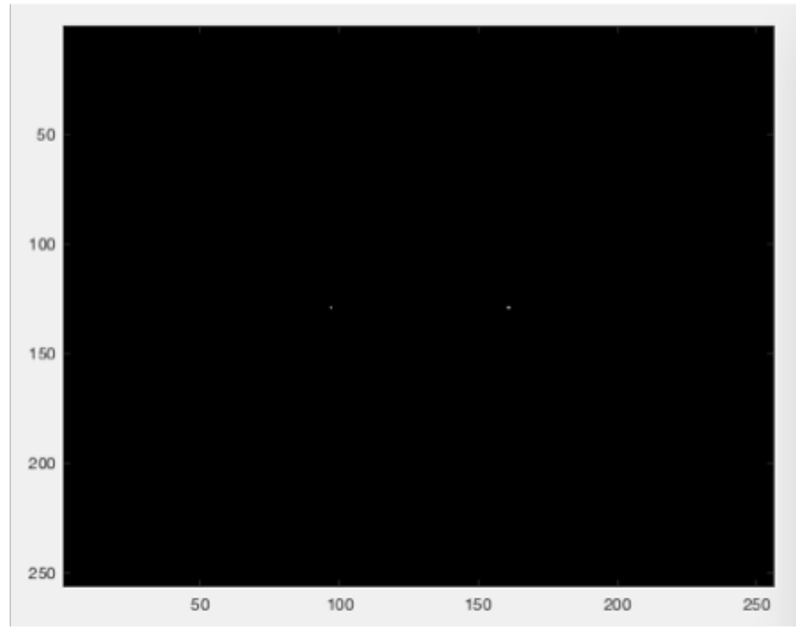
2D Fourier Transform

$$f(x, y) \circ\!\!-\!\!\bullet \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j(\omega_x x + \omega_y y)} dxdy$$

$$F(\omega_x, \omega_y) \bullet\!\!-\!\!\circ \left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\omega_x, \omega_y) e^{j(\omega_x x + \omega_y y)} d\omega_x d\omega_y$$

Example: 1D-cosine as an image





$f(x,y) = \cos(\omega_0 x)$ $\quad$ $f(x,y) \circ\!\!-\!\!\bullet \frac{1}{2}(\delta(\omega_x - \omega_0) + \delta(\omega_x + \omega_0)).\delta(\omega_y)$

Separable functions

$$f(x,y) = f_1(x)f_2(y) \circ\!\!-\!\!\bullet \int_{-\infty}^{\infty} f_1(x)e^{-j\omega_x x}dx \int_{-\infty}^{\infty} f_2(y)e^{-j\omega_y y}dy = F_1(\omega_x)F_2(\omega_y)$$

$$f(x,y) = \cos(\omega_1 x)\cos(\omega_2 y)$$

$$\circ\!\!-\!\!\bullet$$

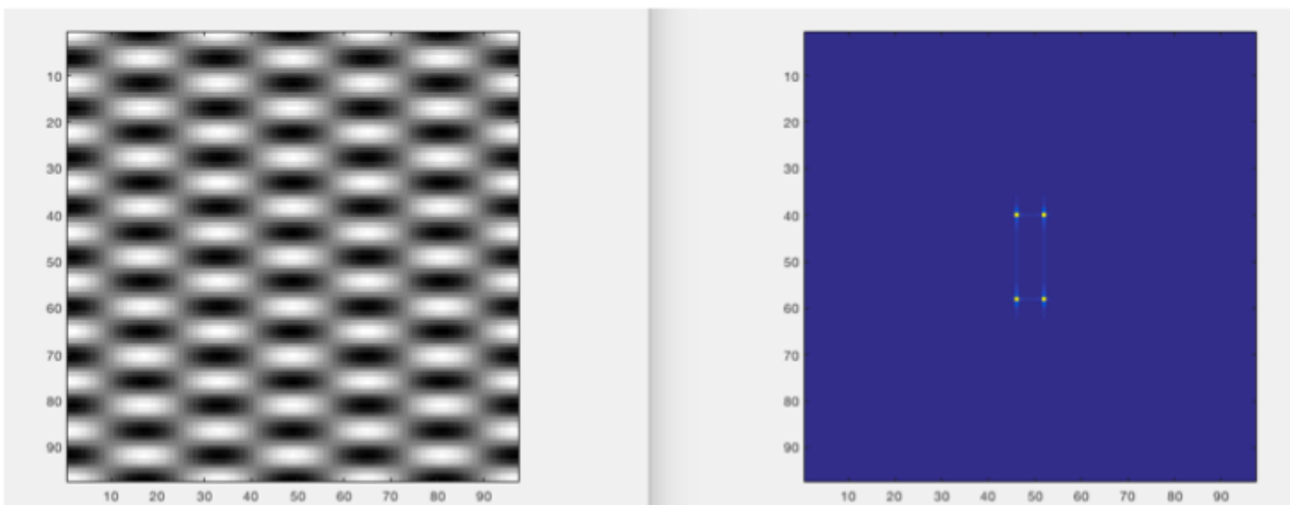
$$\frac{1}{2}(\delta(\omega_x - \omega_1) + \delta(\omega_x + \omega_1))\frac{1}{2}(\delta(\omega_y - \omega_2) + \delta(\omega_y + \omega_2))$$

Separable functions

$$f(x,y) = \cos(\omega_1 x)\cos(\omega_2 y)$$

$$\circ\!\!-\!\!\bullet$$

$$\frac{1}{2}(\delta(\omega_x - \omega_1) + \delta(\omega_x + \omega_1))\frac{1}{2}(\delta(\omega_y - \omega_2) + \delta(\omega_y + \omega_2))$$

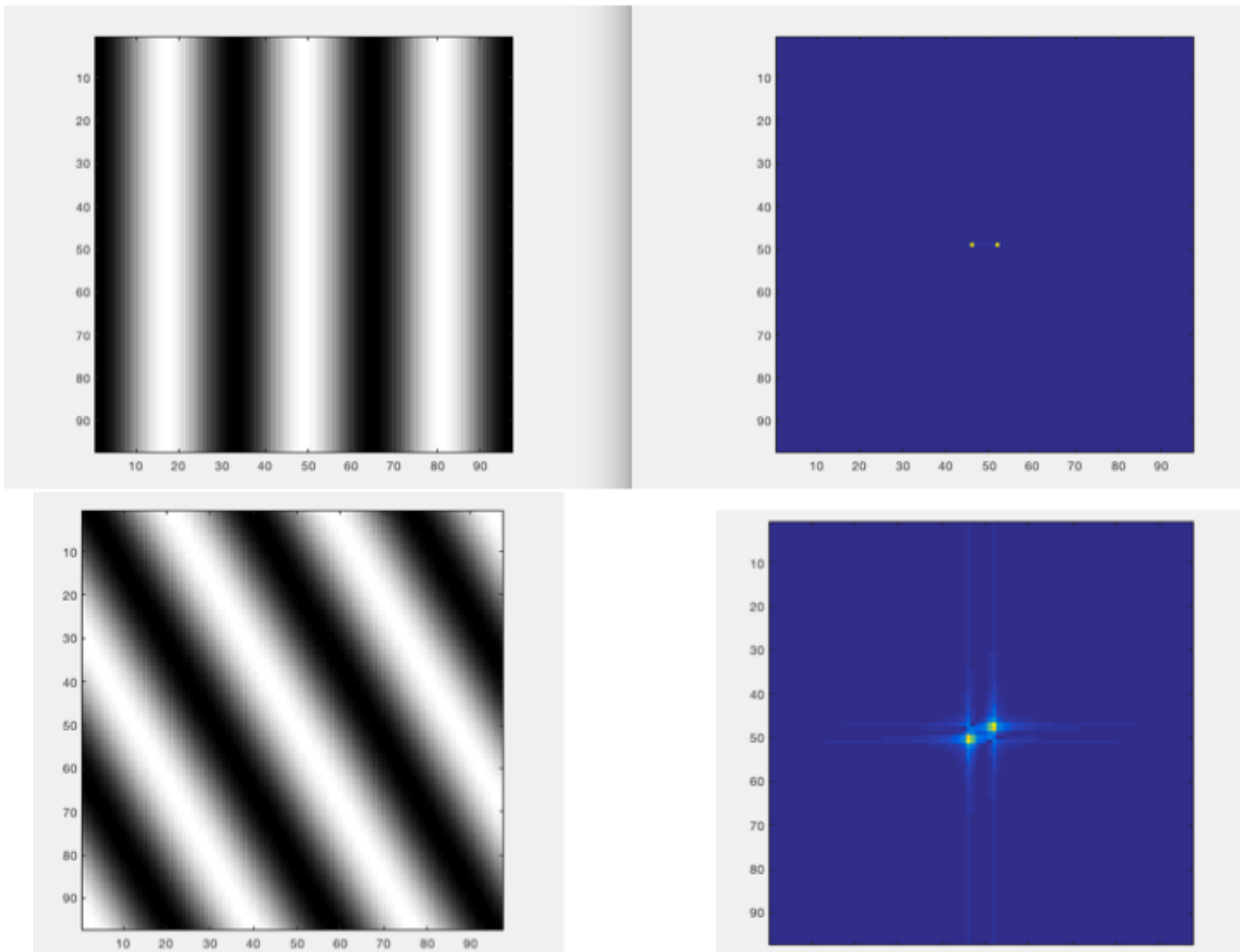


Shift Theorem in 2D

$$f(x-x_0, y-y_0) \circ\!\!-\!\!\bullet\, F(\omega_x, \omega_y)e^{-j(\omega_x x_0+\omega_y y_0)}$$

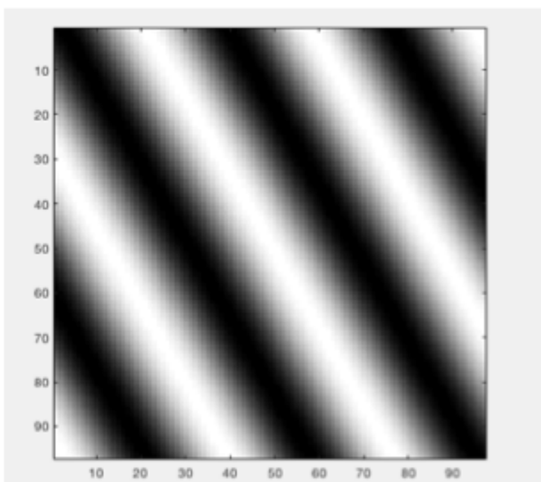
If we know the phases of two 1D signals we can recover their relative displacement?
But can we do that for 2D images?

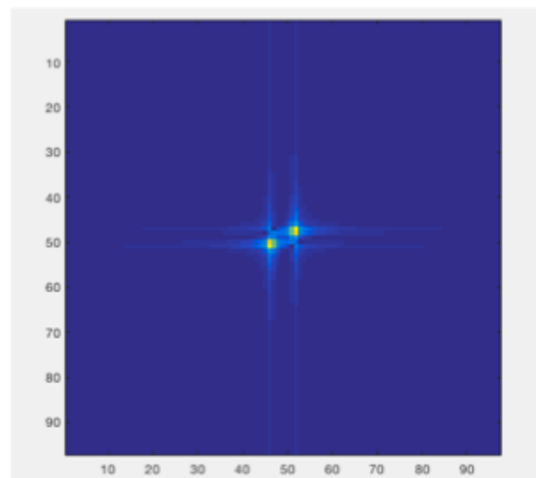
2D rotation


10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90
10 20 30 40 50 60 70 80 90

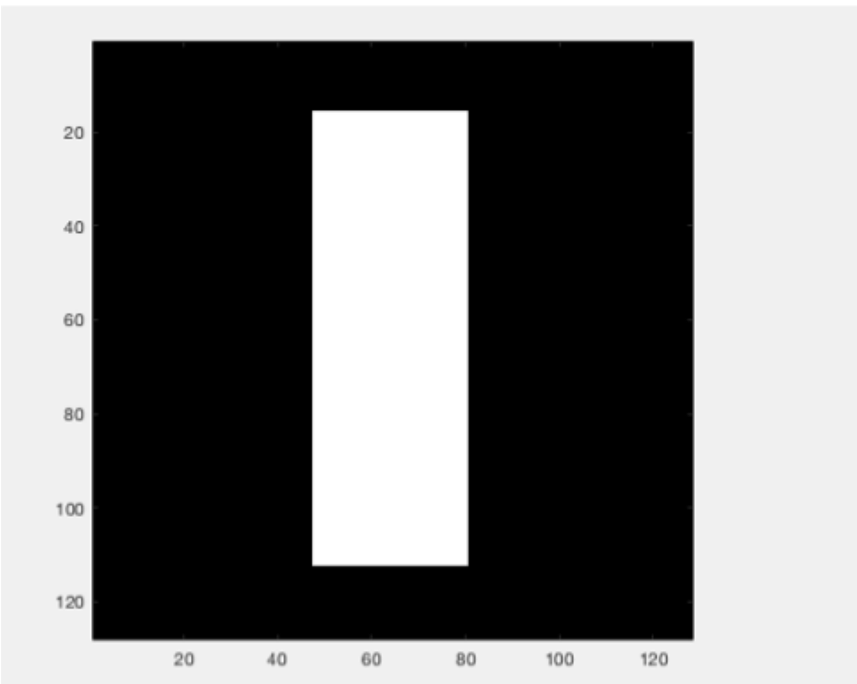
2D rotation

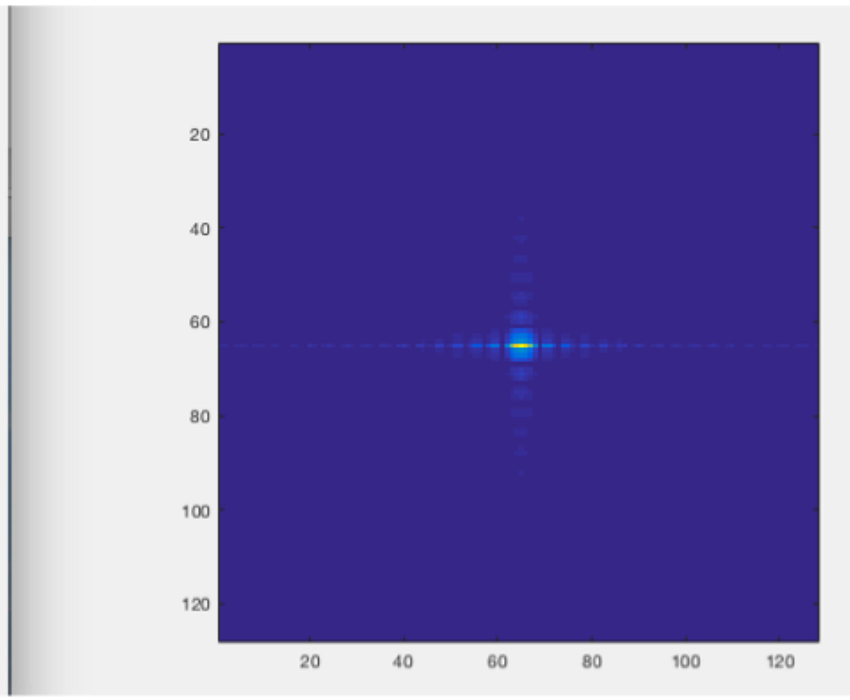
$$f * R\begin{bmatrix} x \\ y \end{bmatrix}) \multimap \mathcal{F}\left(R\begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix}\right)$$



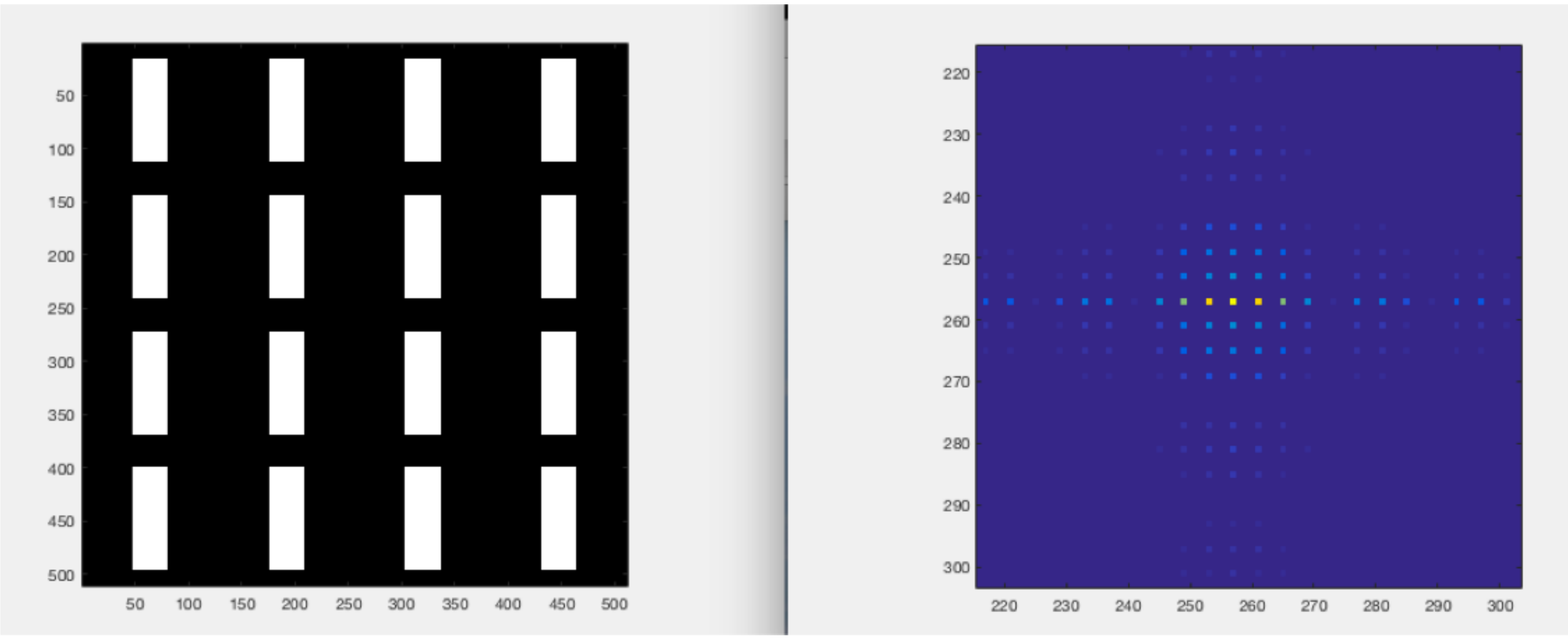


2D Fourier of a box

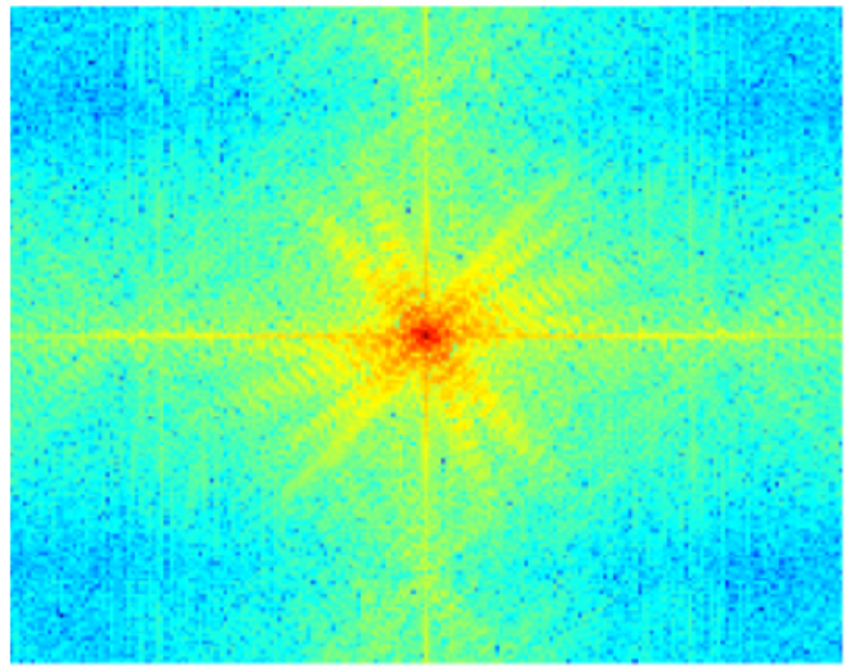




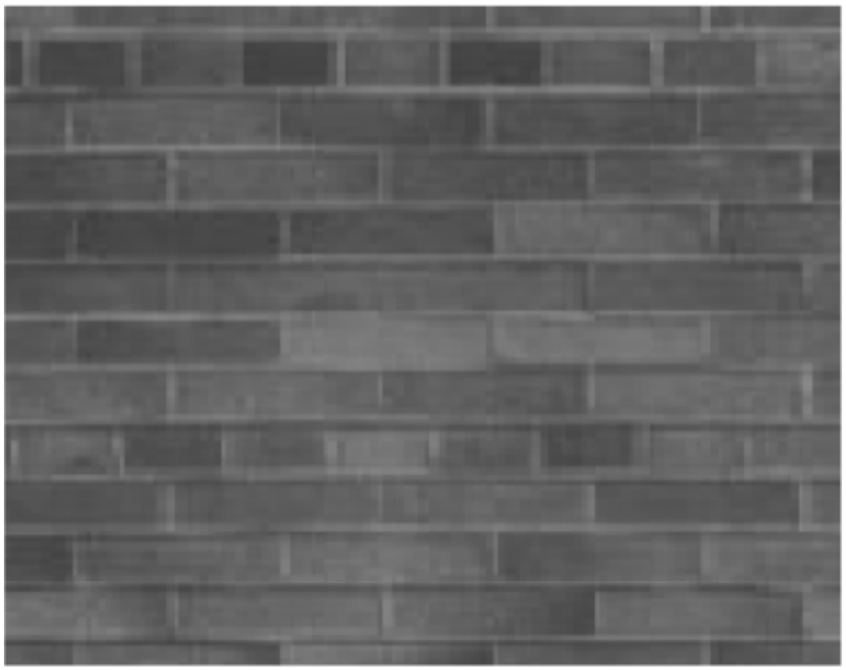
How do we model other periodic patterns?

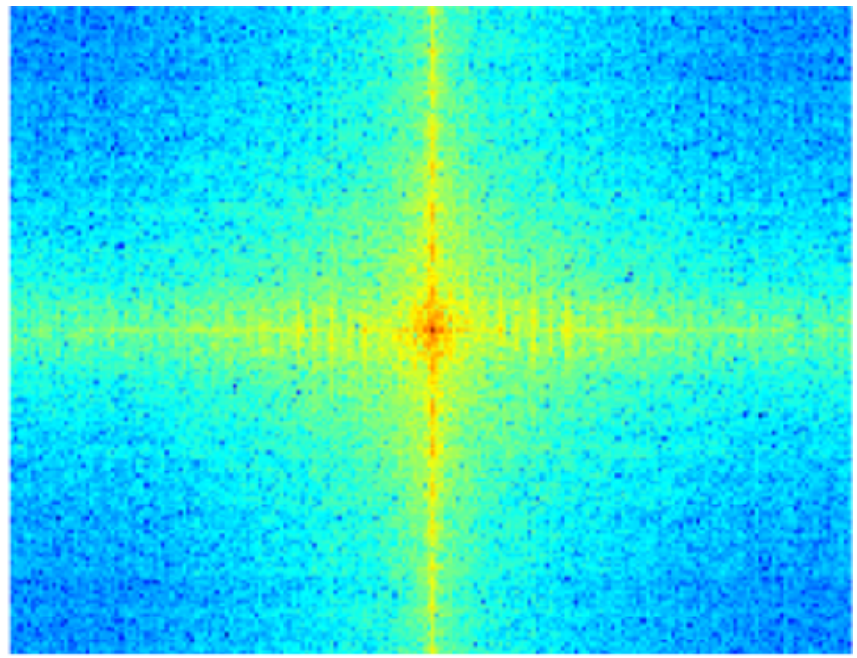


Clue about orientation of edges



Clue about periodicity





Clues about contrast