Camera Projection Matrix

Slides by HyunSoo Park

Raw First-person Footage

Lens configuration (internal parameter)

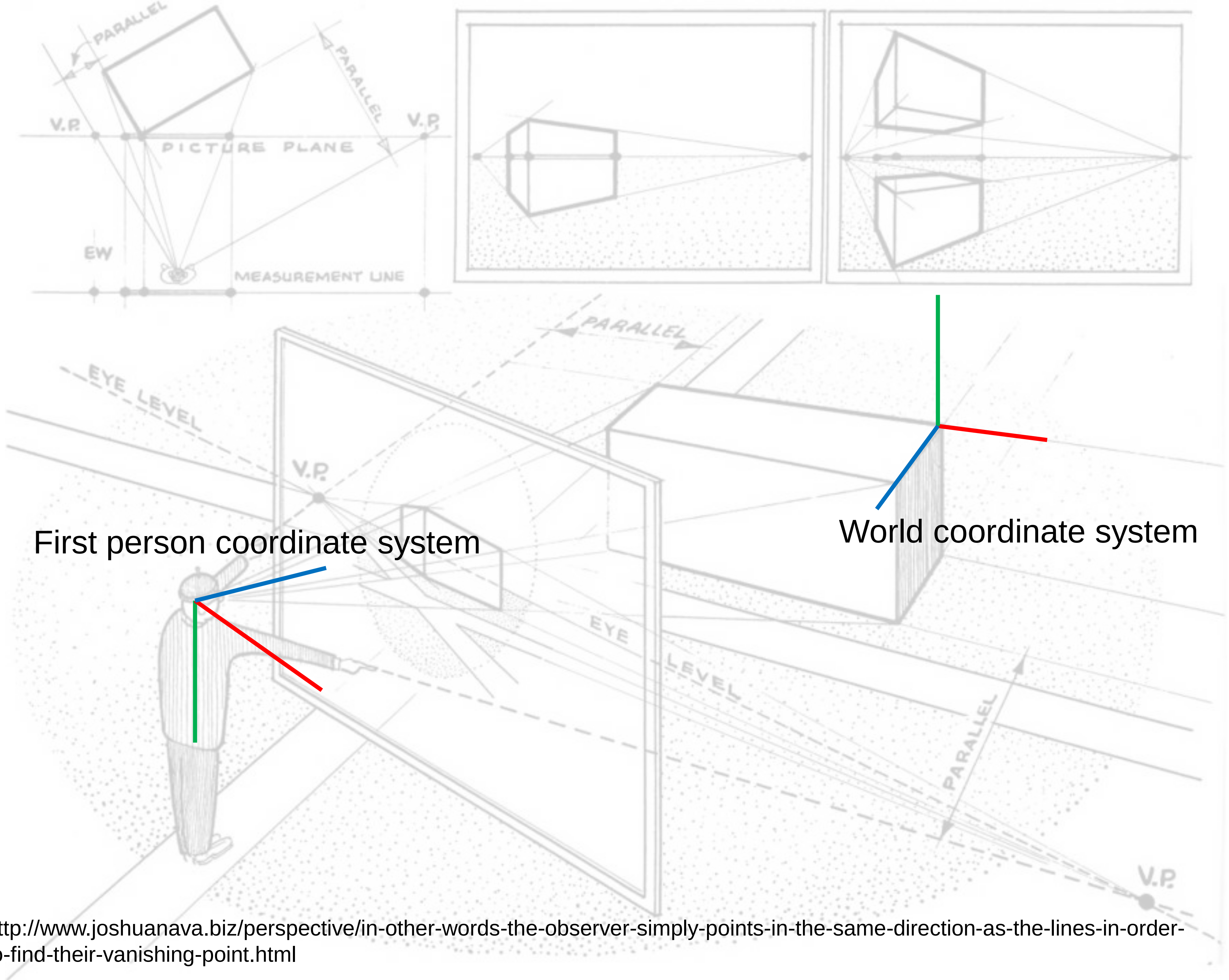
$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = L\left(\mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \right)$$

Spatial relationship between sensor and pinhole (internal parameter)

Camera body configuration (extrinsic parameter)

$$z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_x & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

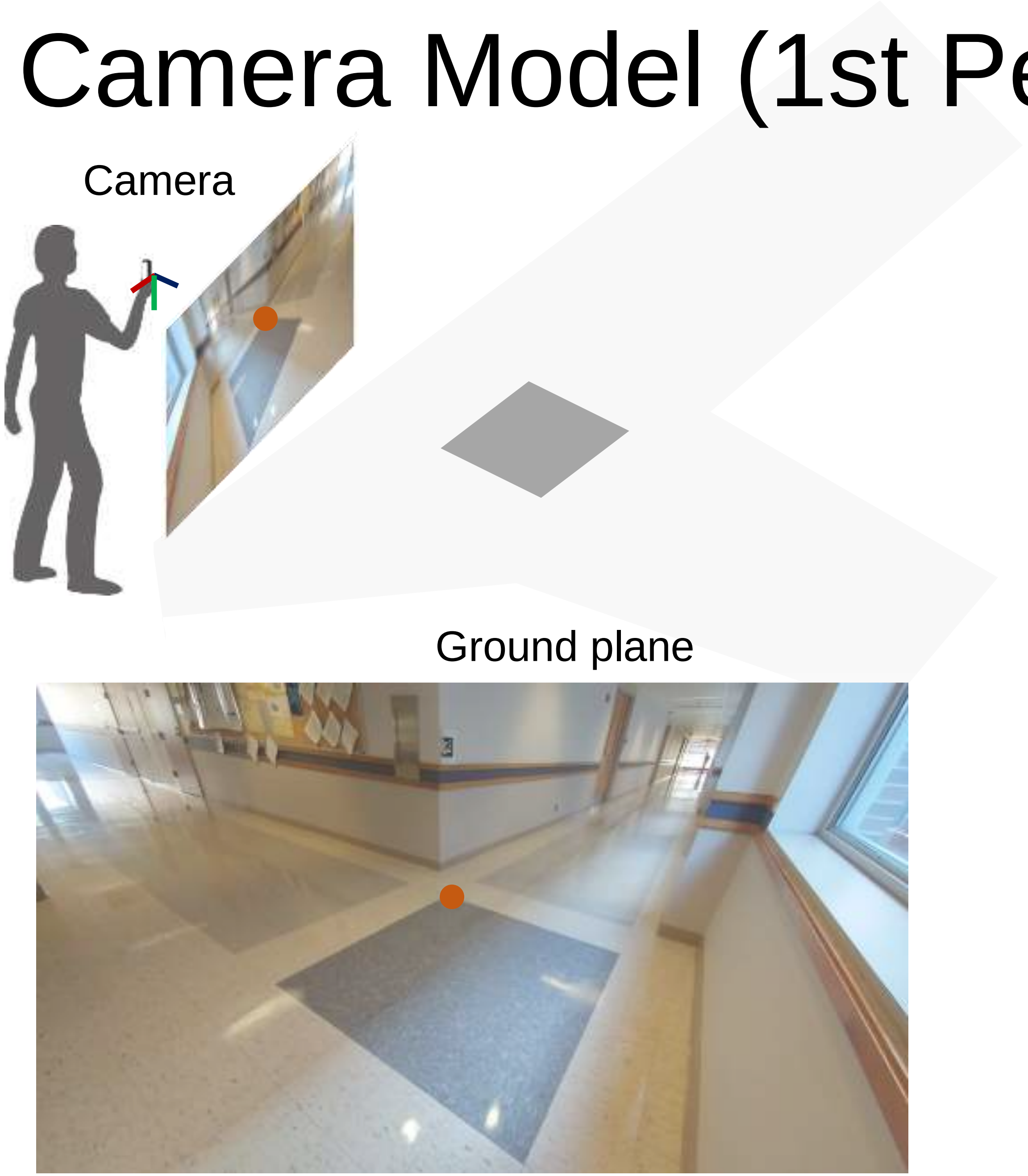
x K $\mathrm{R} \in \mathbb{R}^{3\times 3}$ t X



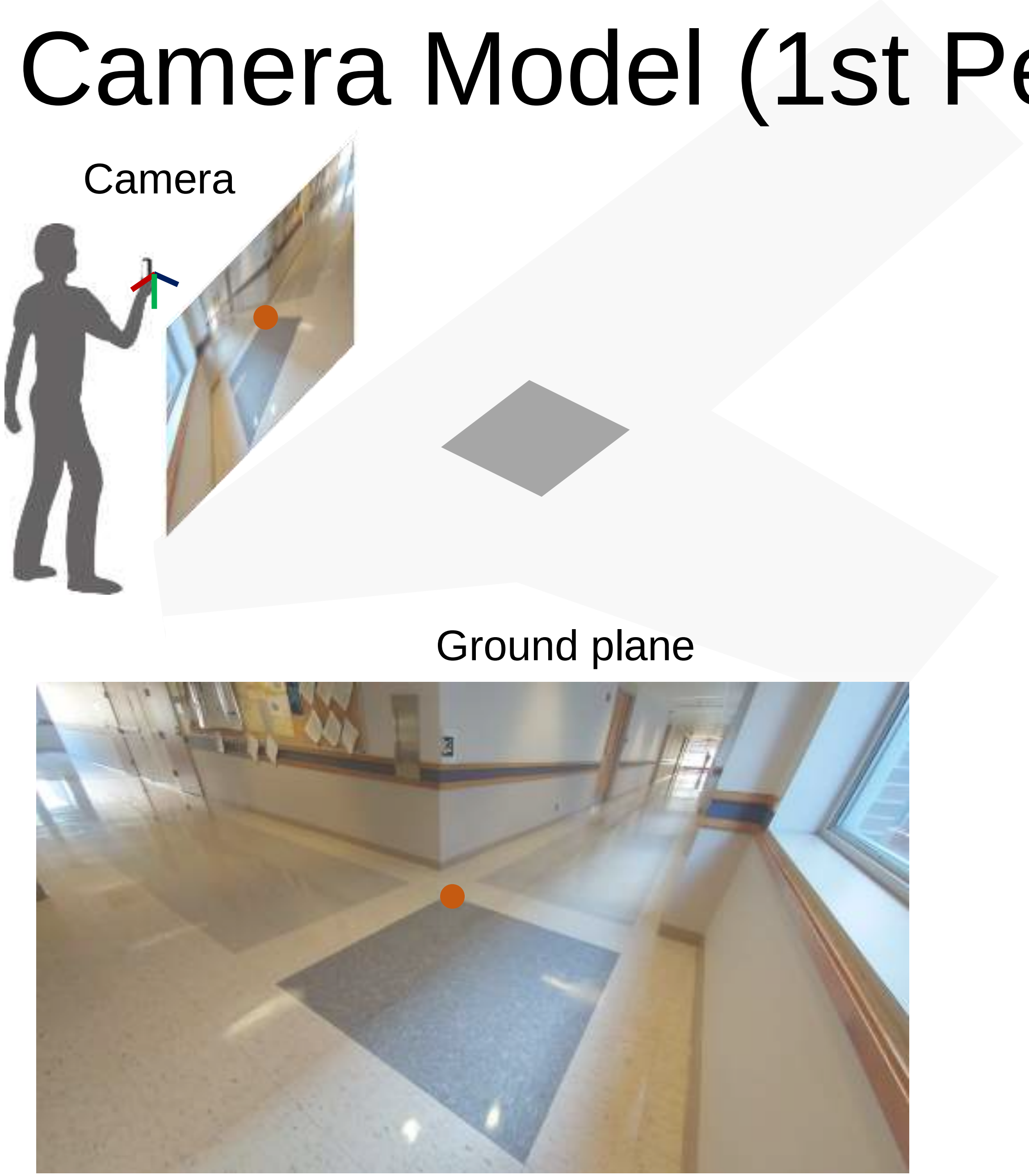
http://www.joshuanava.biz/perspective/in-other-words-the-observer-simply-points-in-the-same-direction-as-the-lines-in-order-to-find-their-vanishing-point.html

Camera Model

Camera Model (1st Person Perspective)

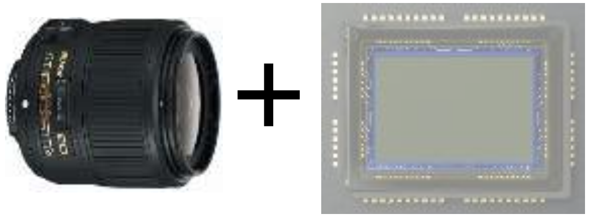


Camera Model (1st Person Perspective)



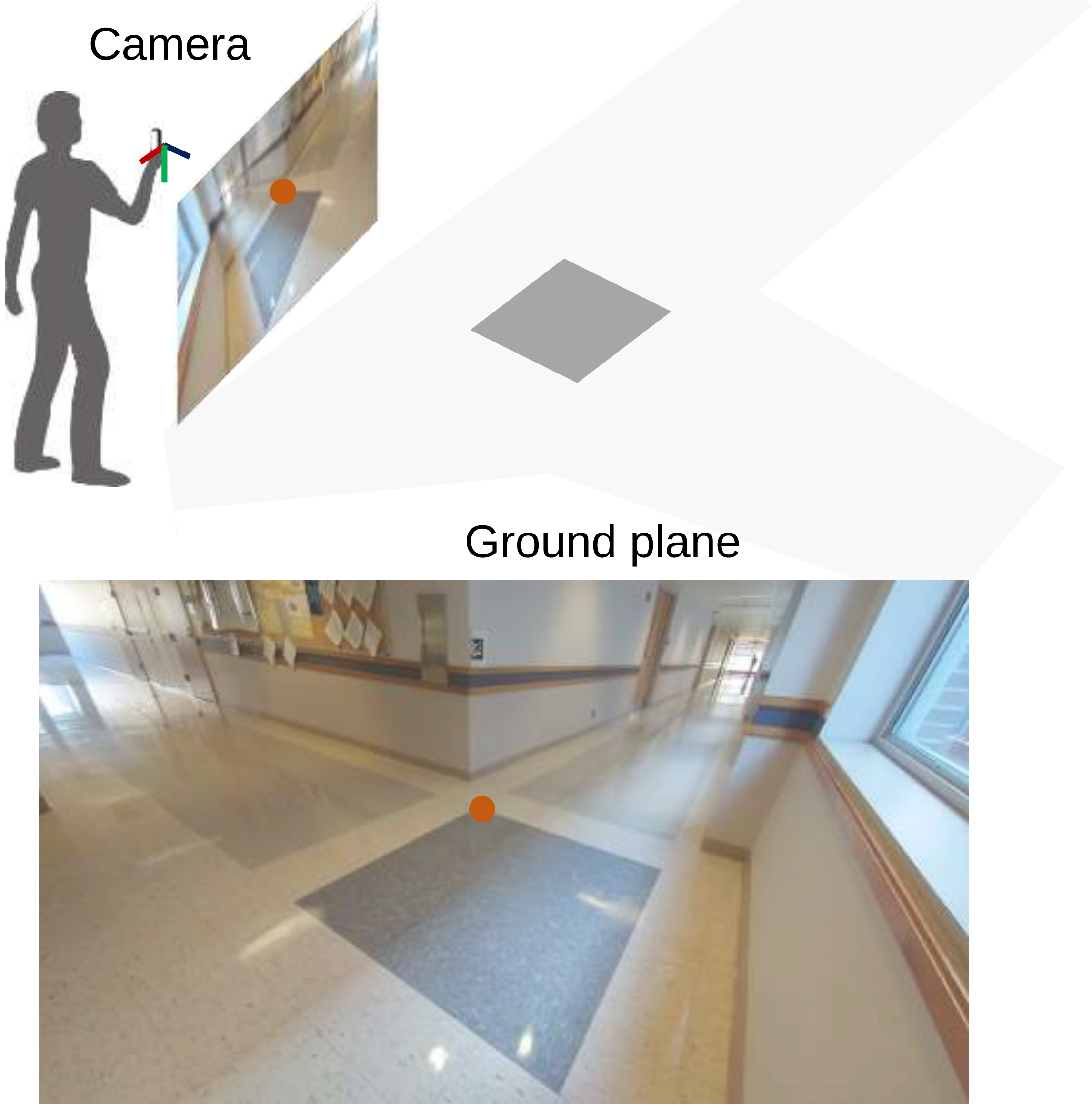
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



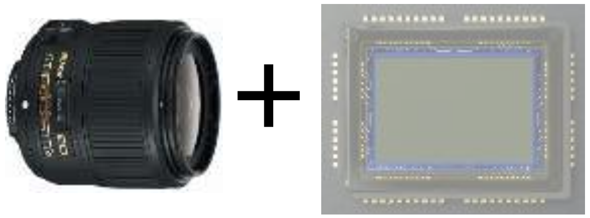
Camera intrinsic parameter
: metric space to pixel space

Camera Model (1st Person Perspective)



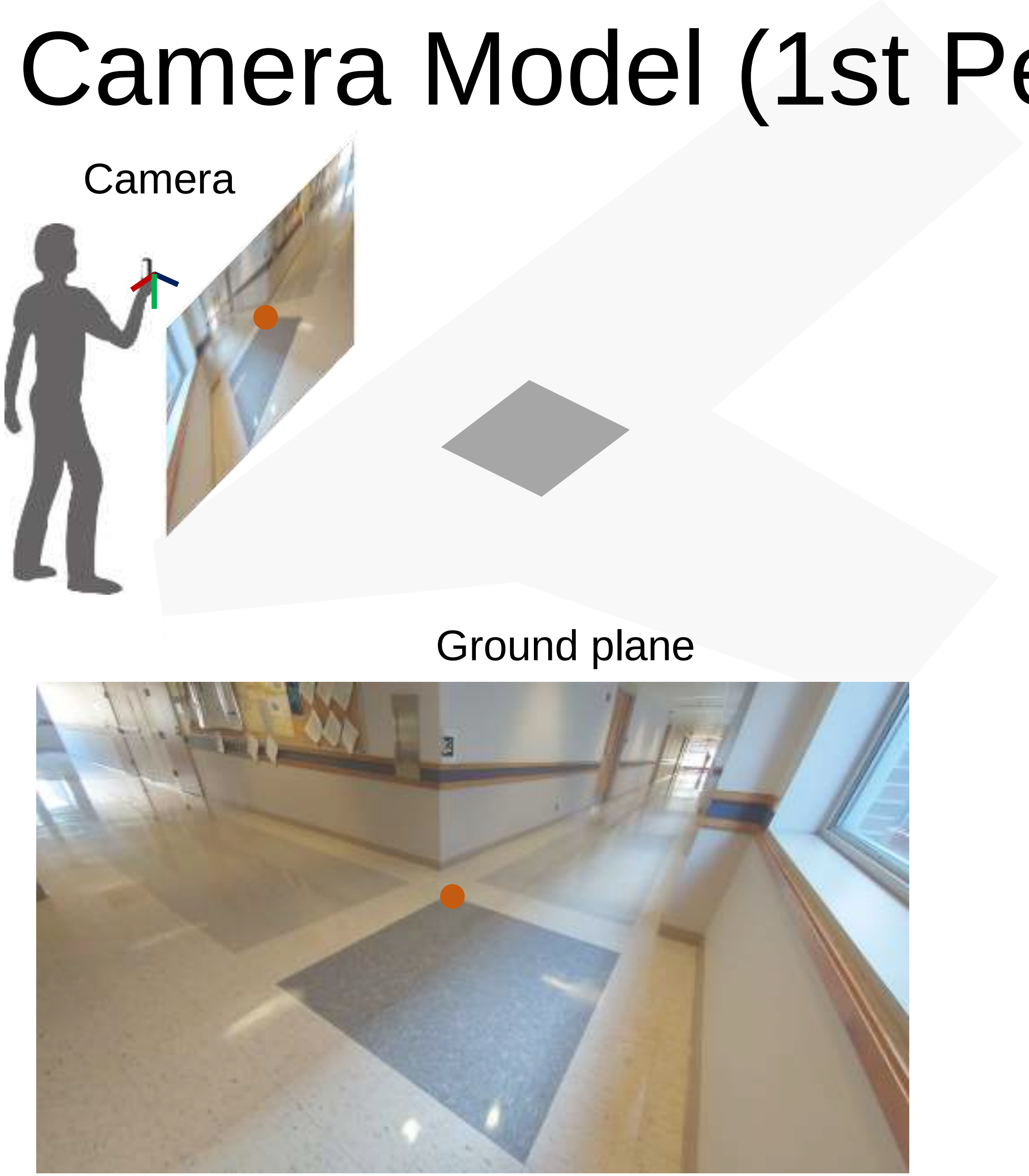
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter
: metric space to pixel space

Camera Model (1st Person Perspective)

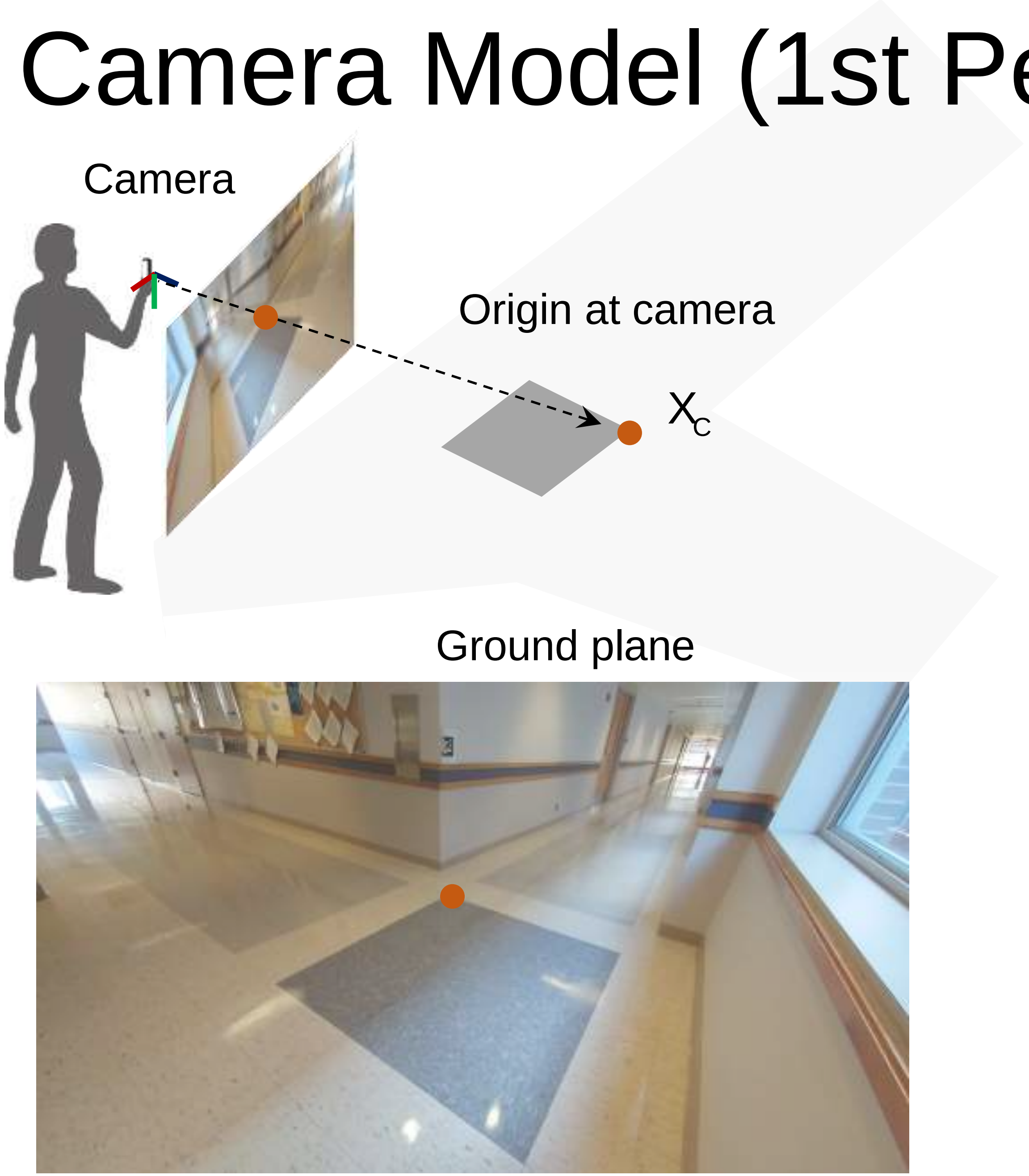


Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad \mathrm{K} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

2D image (pix) 3D world (metric)

Camera Model (1st Person Perspective)



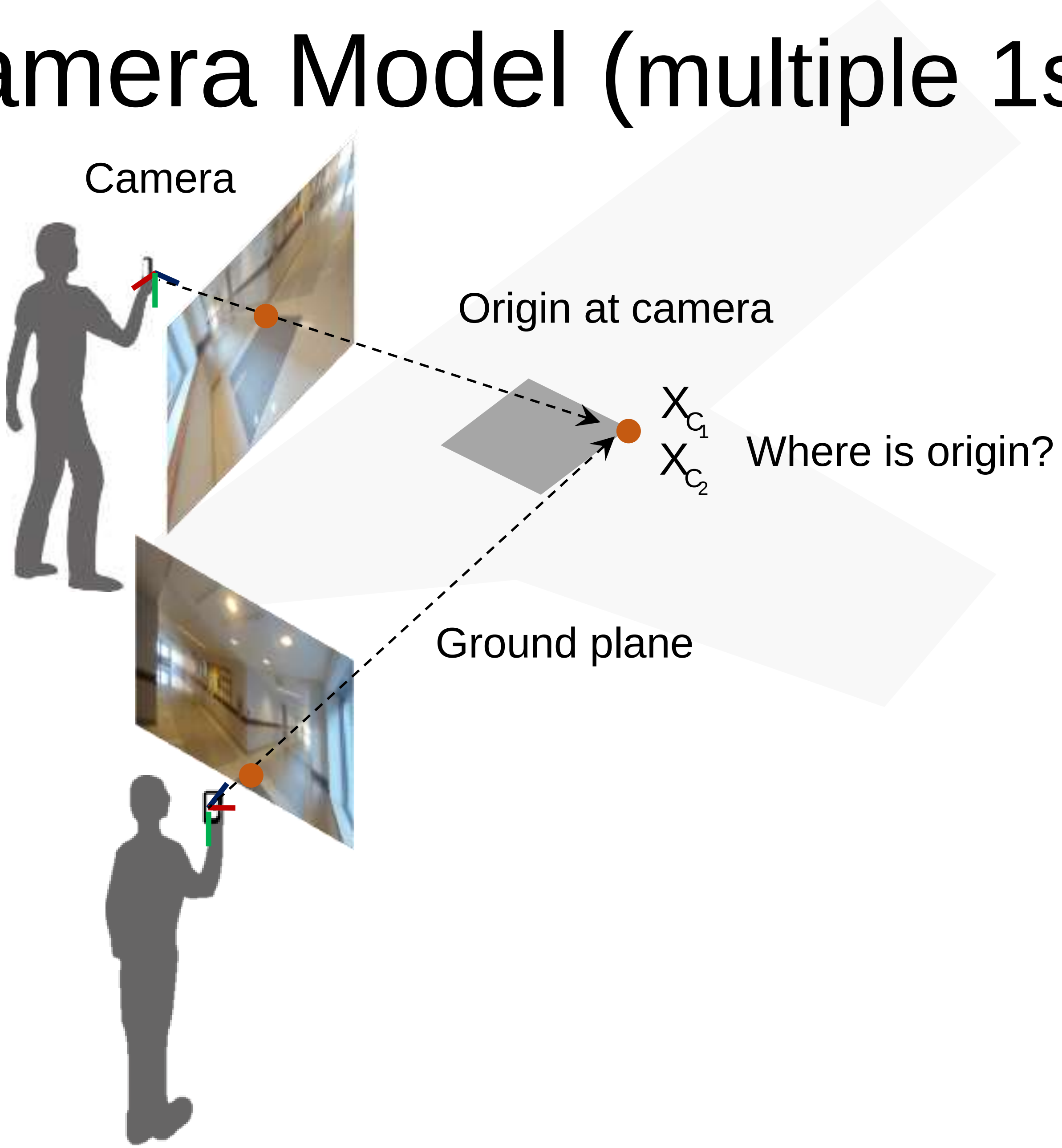
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad \mathrm{K} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

2D image (pix) 3D world (metric)

$$\longrightarrow \lambda \mathrm{K}^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathrm{X}_{\mathrm{C}}$$

Camera Model (multiple 1st Person Perspective)



Recall camera projection matrix:

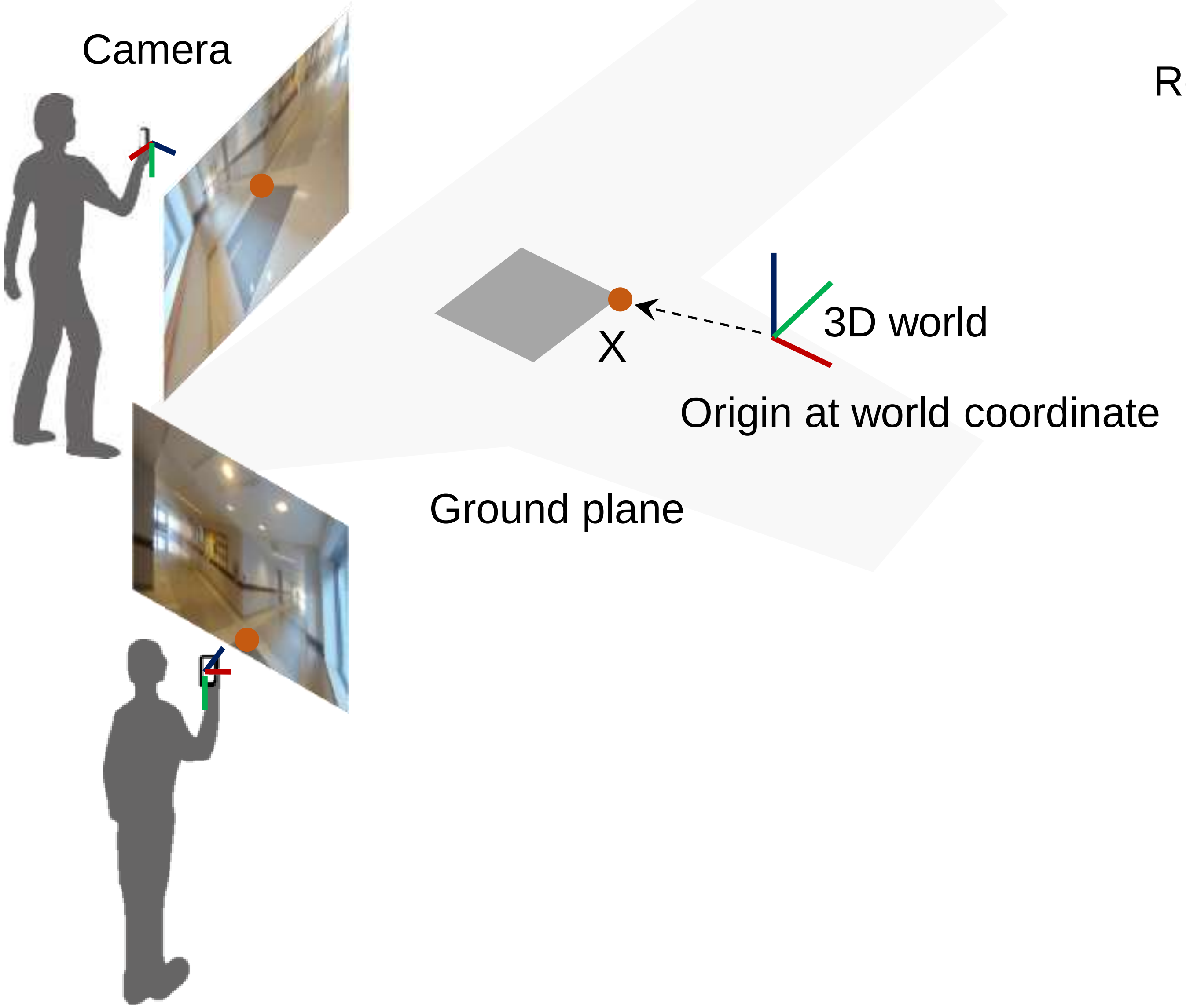
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2D image (pix) 3D world (metric)

$$\longrightarrow \lambda \mathrm{K}^{-1} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \mathrm{X}_{C_1}$$

$$\lambda \mathrm{K}^{-1} \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathrm{X}_{C_2}$$

Camera Model (3rd Person Perspective)



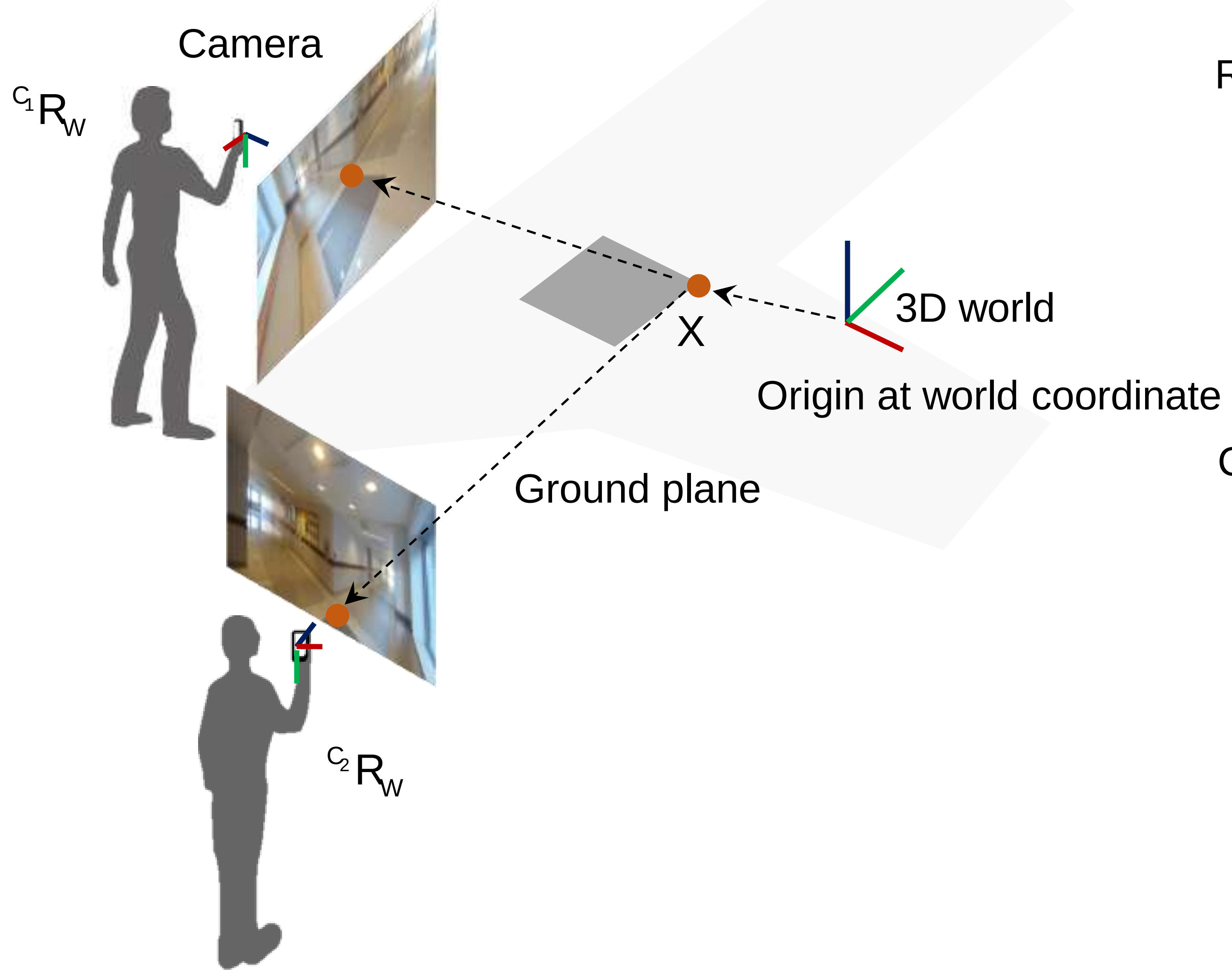
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K} \, \mathrm{X}$$

2D image (pix) 3D world (metric)

Coordinate Transform (Rotation matrix)



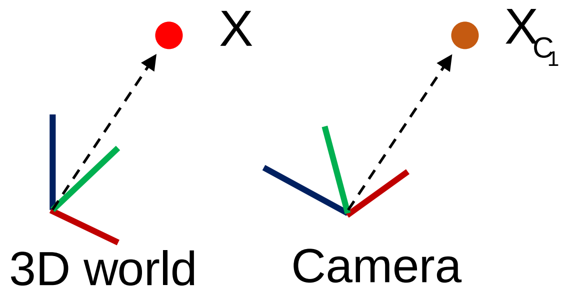
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}}_{\mathrm{X}}$$

2D image (pix) 3D world (metric)

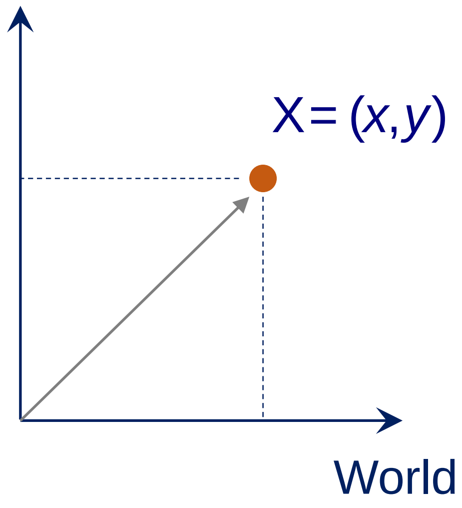
Coordinate transformation from world to camera:

$$\mathrm{X}_{C_1} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \mathrm{X}$$



Coordinate Transform (Rotation)

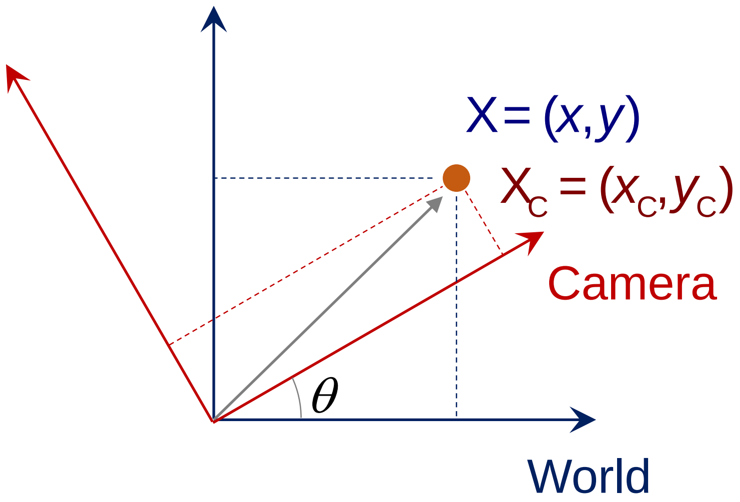
2D coordinate transform:



$$\begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transform (Rotation)

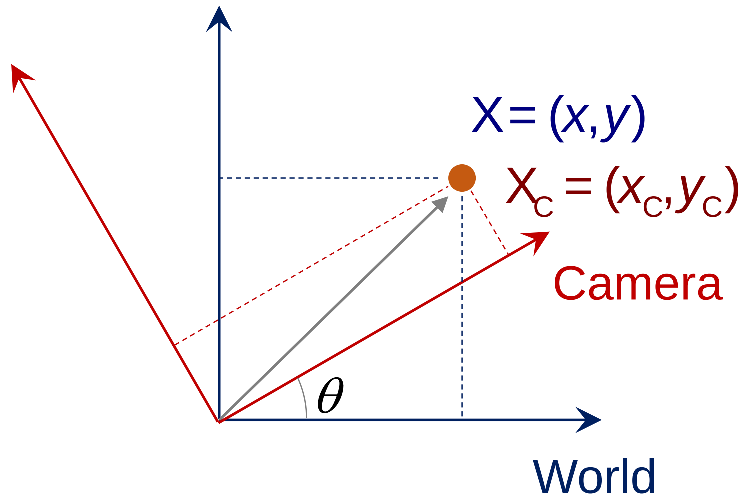
2D coordinate transform:



$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \quad ? \quad \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transform (Rotation)

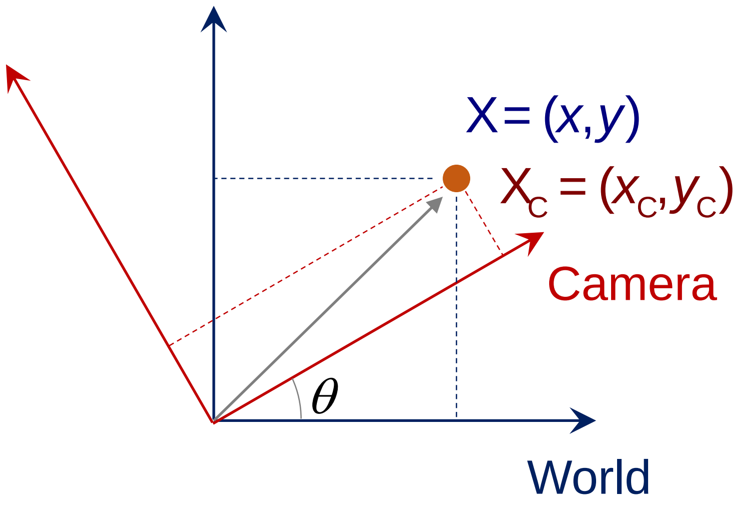
2D coordinate transform:



$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transform (Rotation)

2D coordinate transform:

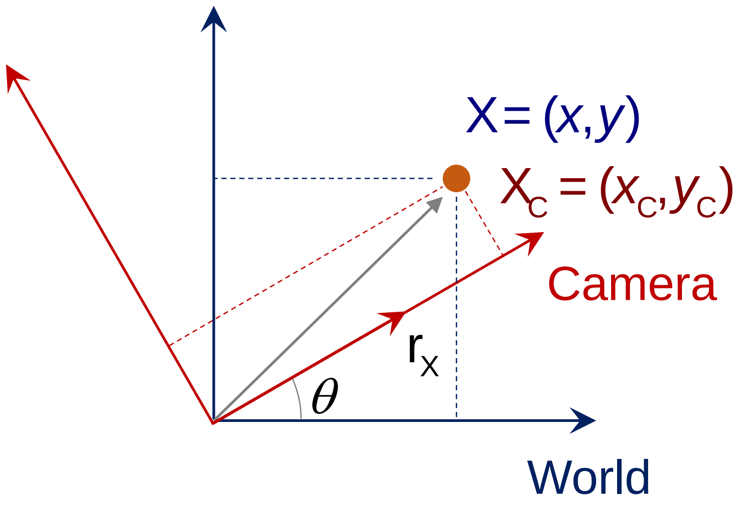


$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\det\left(\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \right) = \cos^2\theta + \sin^2\theta = 1$$

Coordinate Transform (Rotation)

2D coordinate transform:

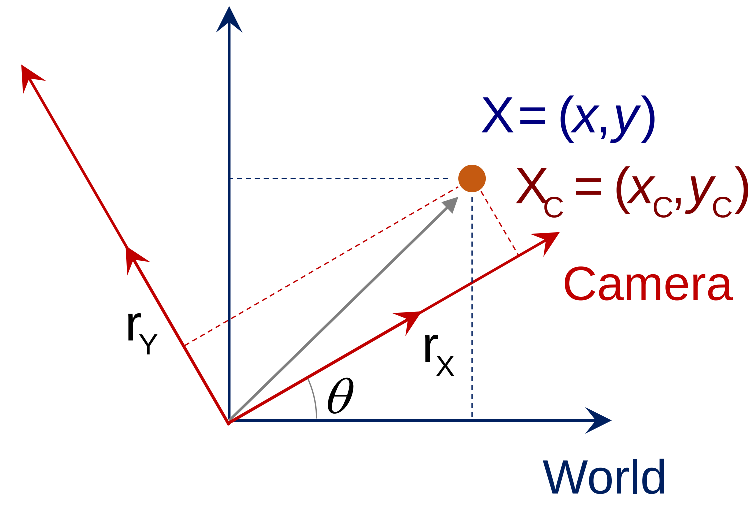


$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \begin{bmatrix} \cos\theta \; r_X \; \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

r_X : x axis of camera seen from the world

Coordinate Transform (Rotation)

2D coordinate transform:



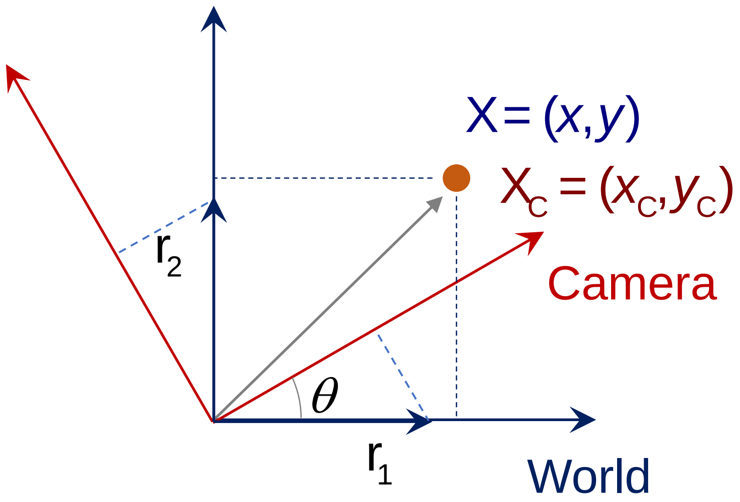
$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \begin{bmatrix} \cos\theta \;\; r_X \;\; \sin\theta \\ -\sin\theta \;\; r_Y \;\; \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

r_X : x axis of camera seen from the world

r_Y : y axis of camera seen from the world

Coordinate Transform (Rotation)

2D coordinate transform:



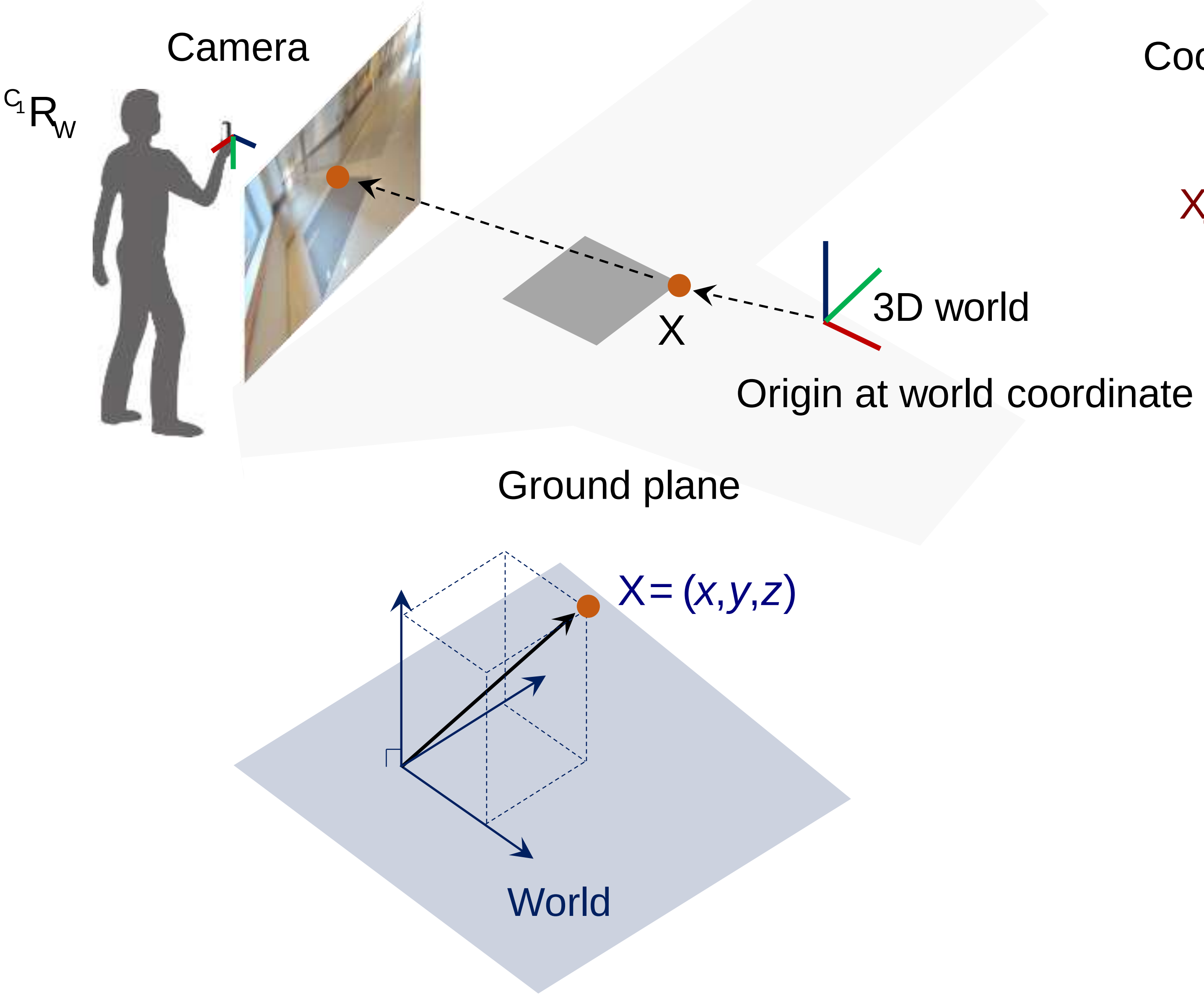
$$\begin{bmatrix} x_C \\ y_C \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(columns: r_1 = first column, r_2 = second column)

r_1 : x axis of world seen from the camera

r_2 : y axis of world seen from the camera

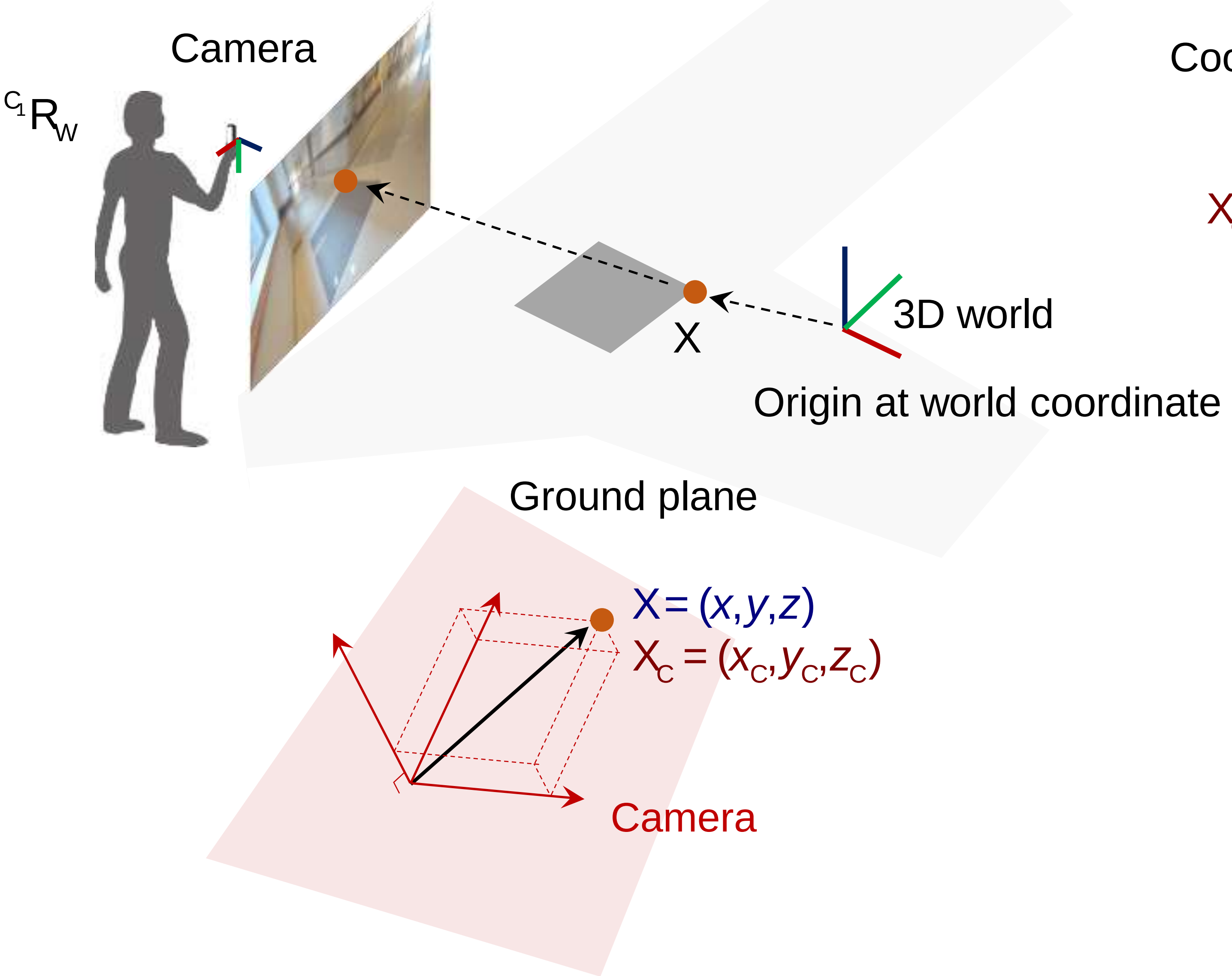
Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

X_C = ? X

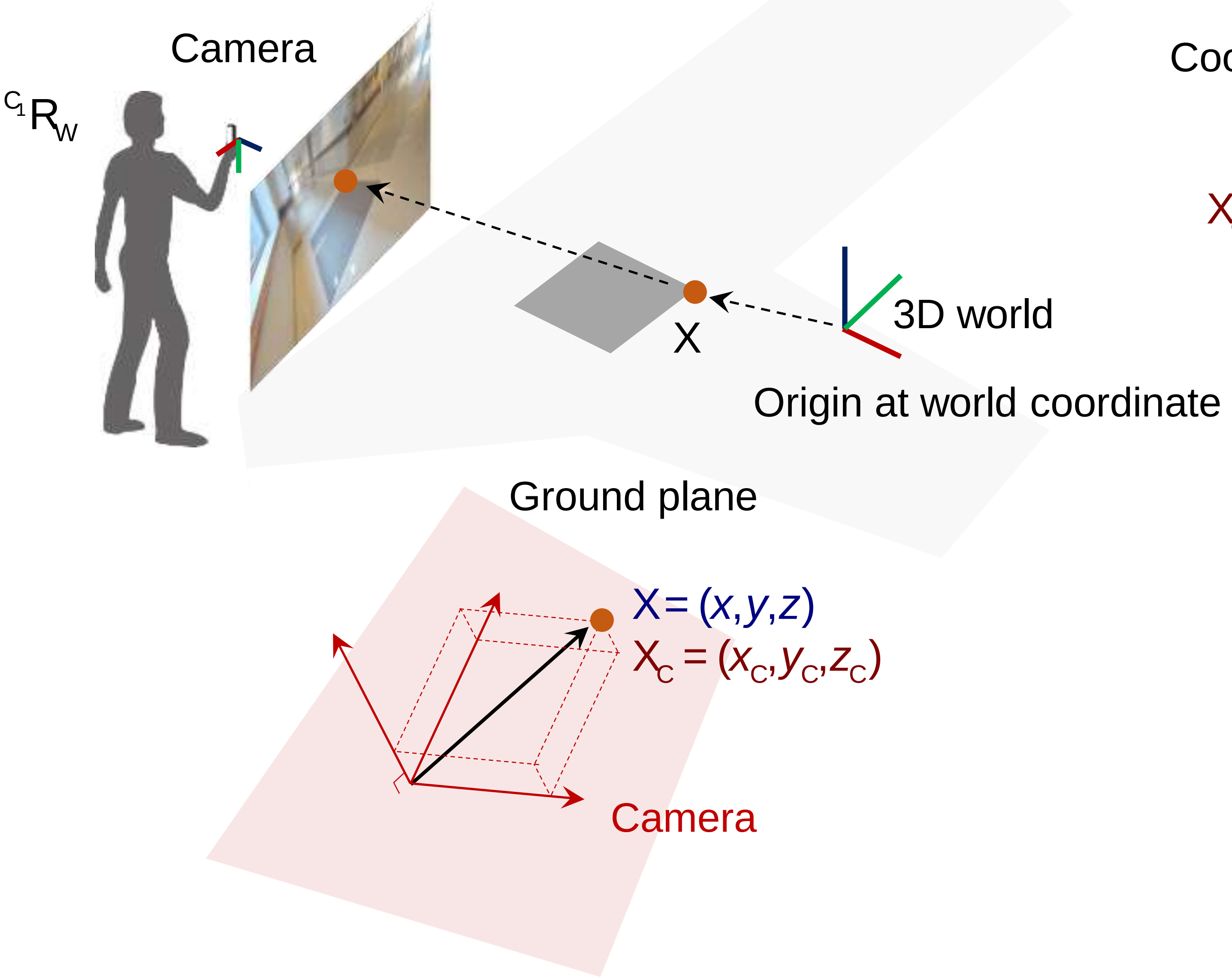
Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

$X_C =$? X

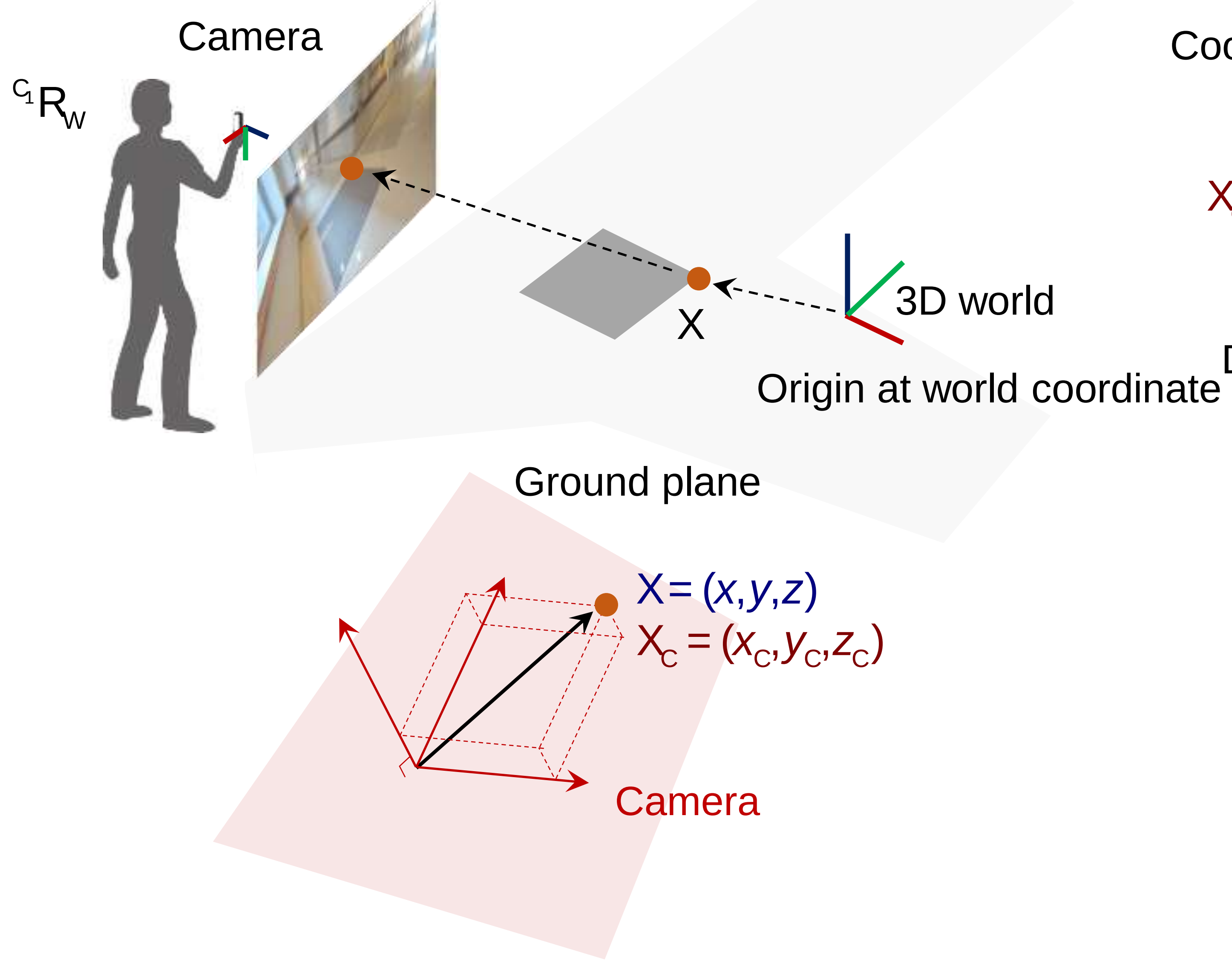
Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

$$\mathbf{X}_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \mathbf{X} = {}^{C}\mathbf{R}_{W}\mathbf{X}$$

Coordinate Transform (Rotation)

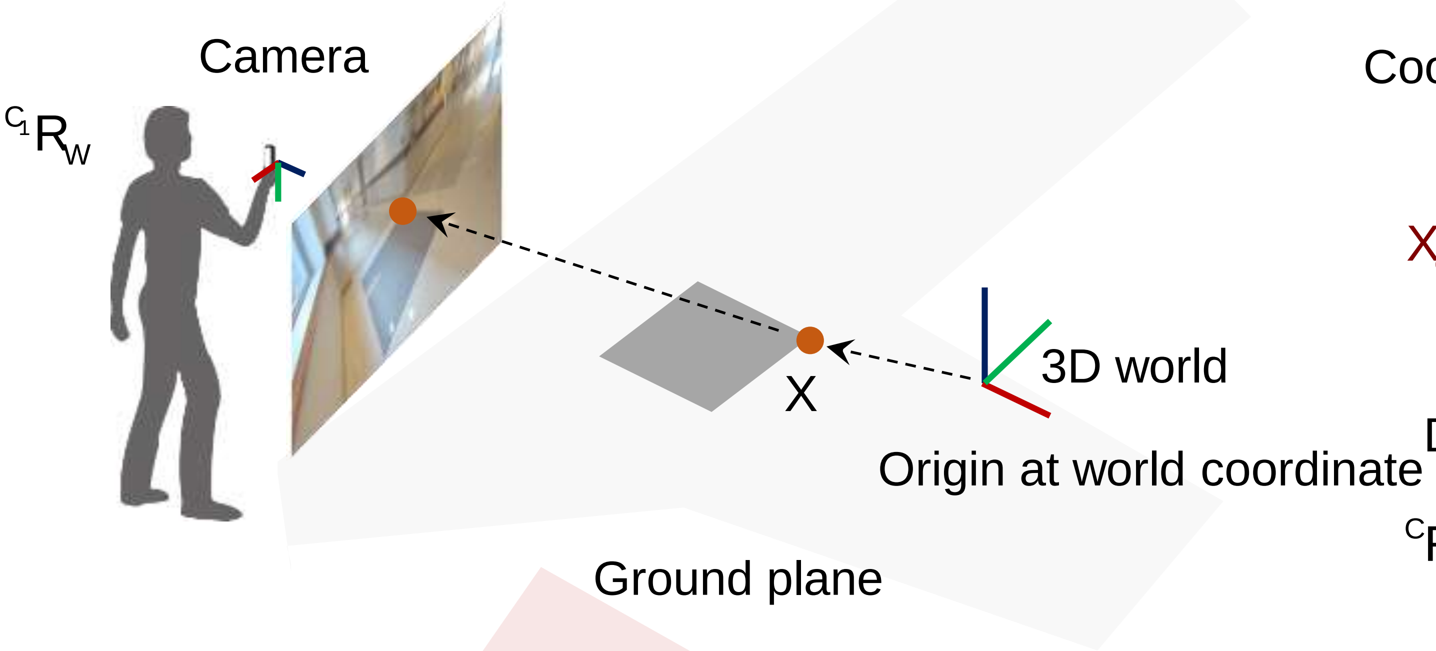


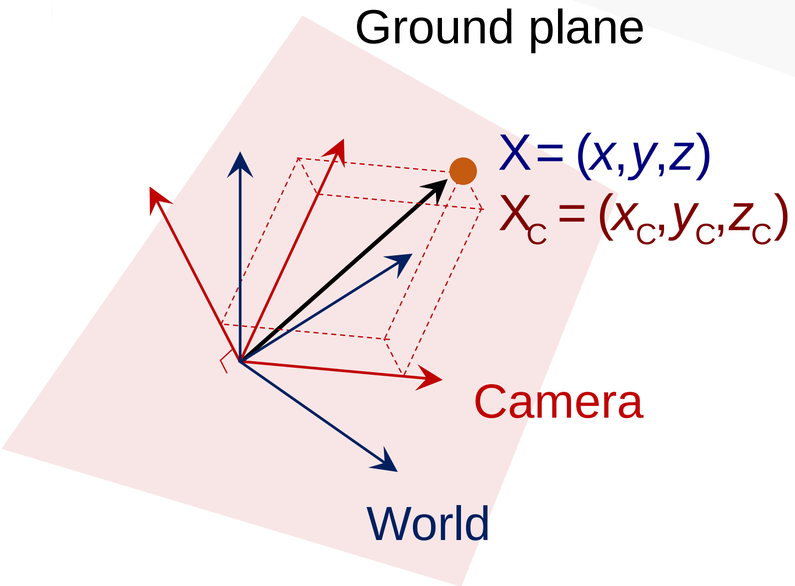
Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X$$

Degree of freedom?

Coordinate Transform (Rotation)





Coordinate transformation from world to camera:

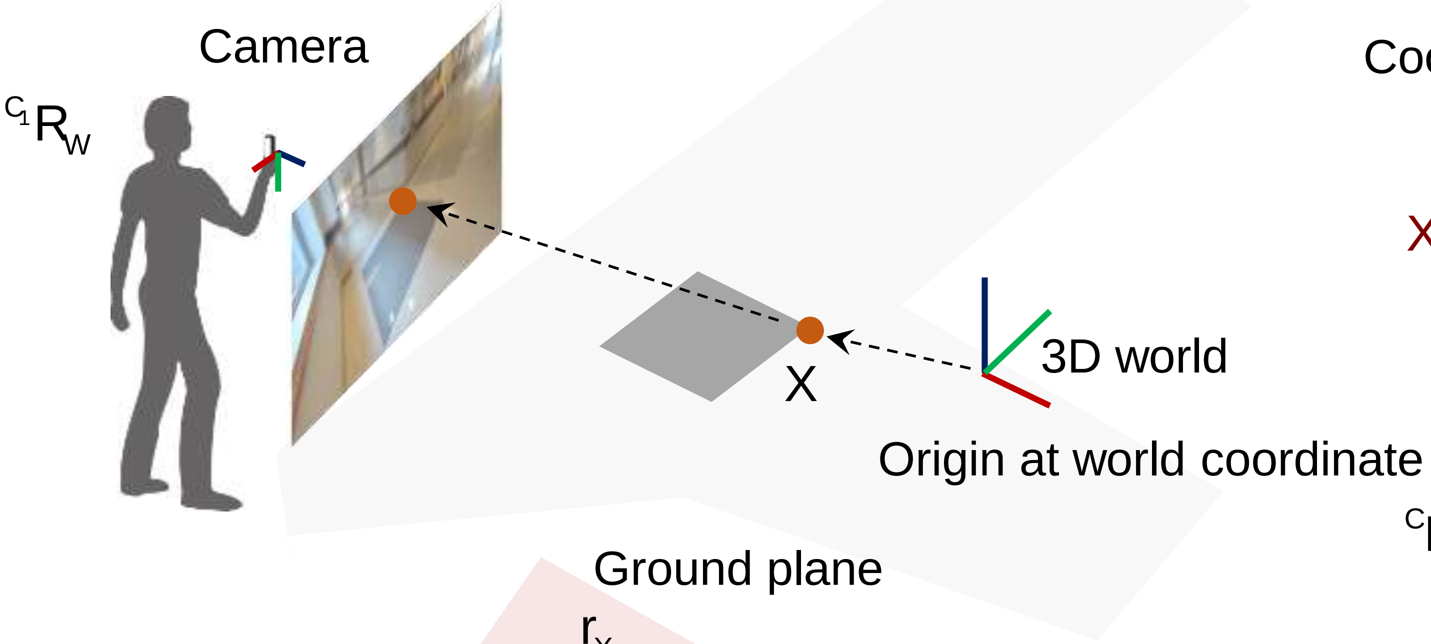
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_{W} X$$

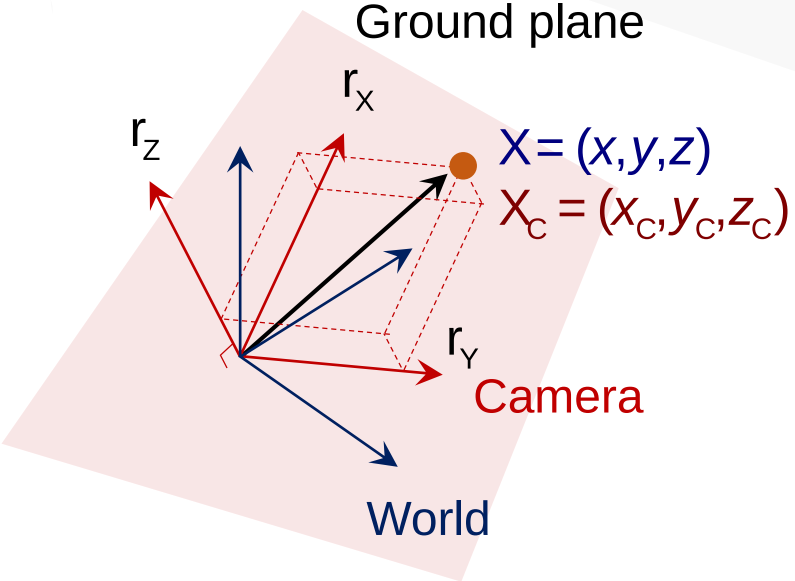
Degree of freedom?

$${}^{C}R_{W} \in SO(3)$$

- Orthogonal matrix → $\left({}^{C}R_{W}\right)^{T}\left({}^{C}R_{W}\right) = I_3, \quad \det\left({}^{C}R_{W}\right) = 1$

Coordinate Transform (Rotation)





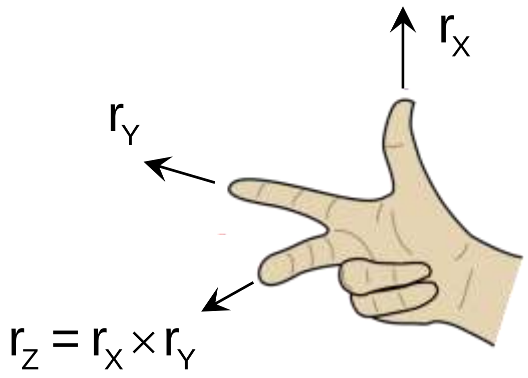
Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X$$

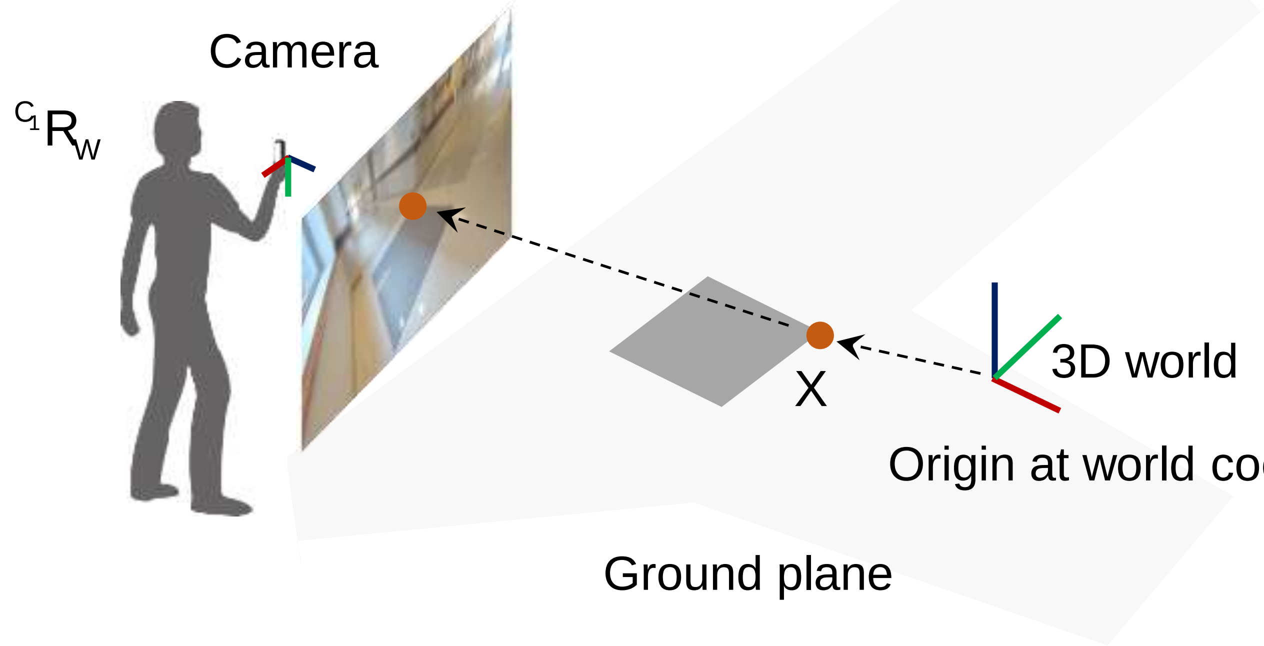
(rows: r_X, r_Y, r_Z)

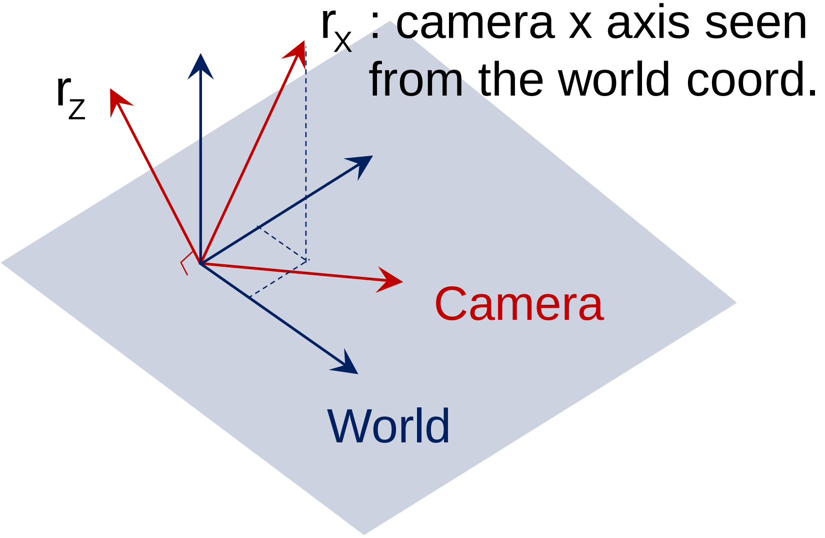
$${}^{C}R_W \in SO(3)$$

- Orthogonal matrix → $\left({}^{C}R_W\right)^T\left({}^{C}R_W\right) = I_3, \quad \det\left({}^{C}R_W\right) = 1$
- Right hand rule



Coordinate Transform (Rotation)





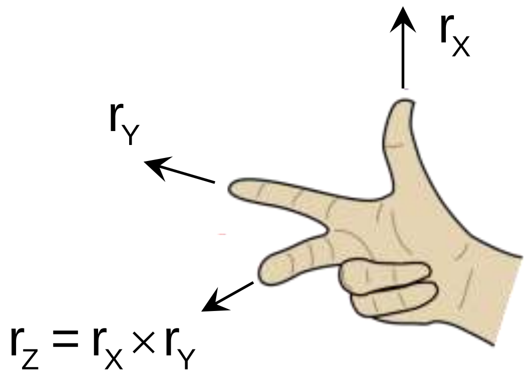
Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^C R_W X$$

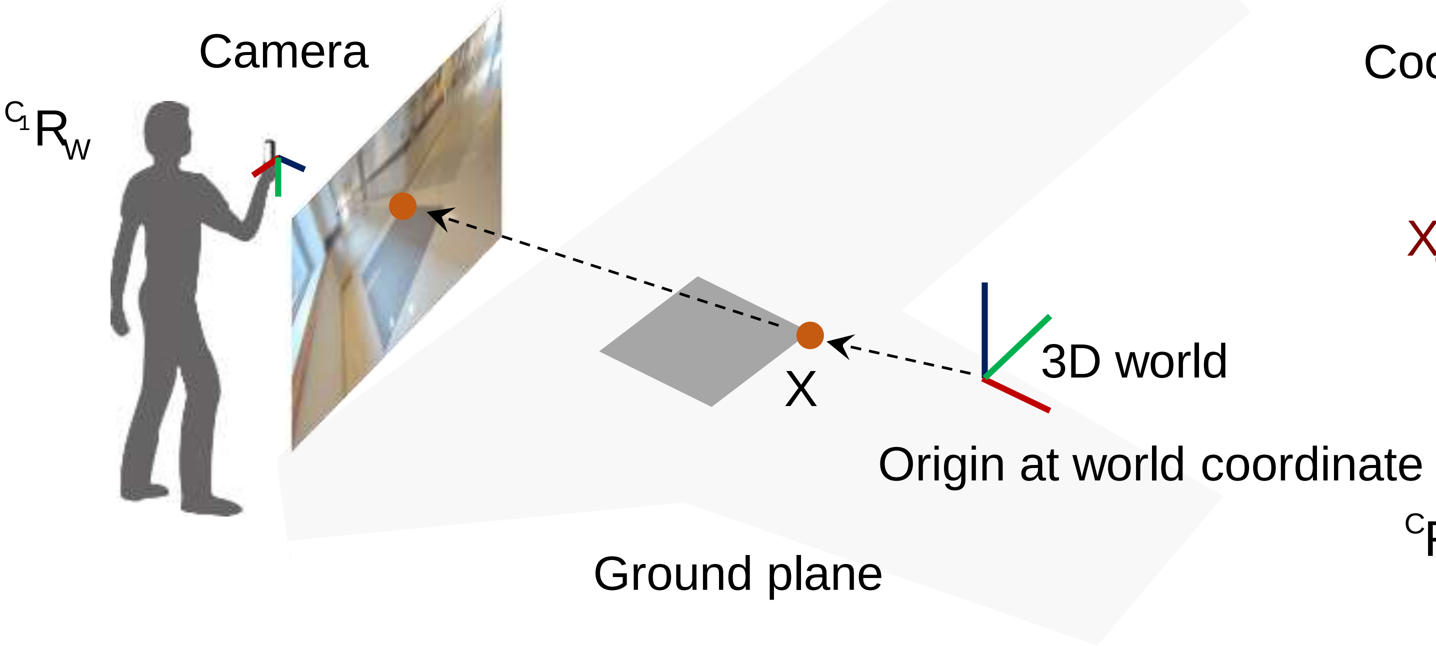
(rows: r_X, r_Y, r_Z)

$^C R_W \in SO(3)$

- Orthogonal matrix → $\left({}^C R_W\right)^T \left({}^C R_W\right) = I_3, \quad \det\left({}^C R_W\right) = 1$
- Right hand rule



Coordinate Transform (Rotation)

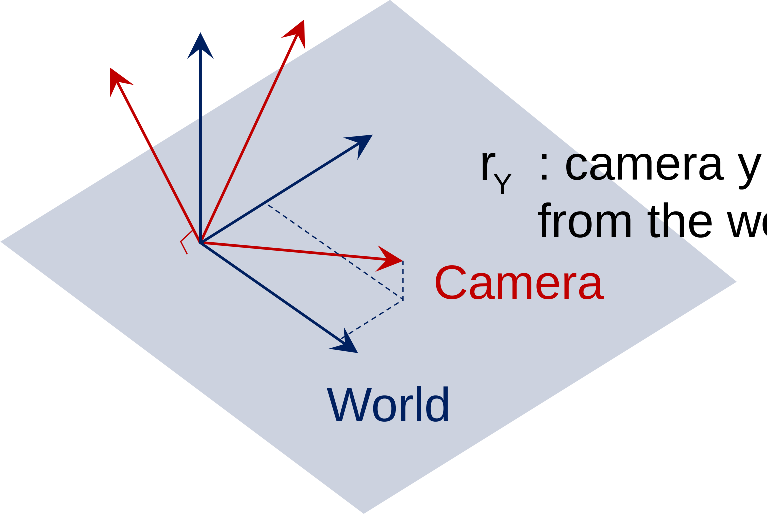


Coordinate transformation from world to camera:

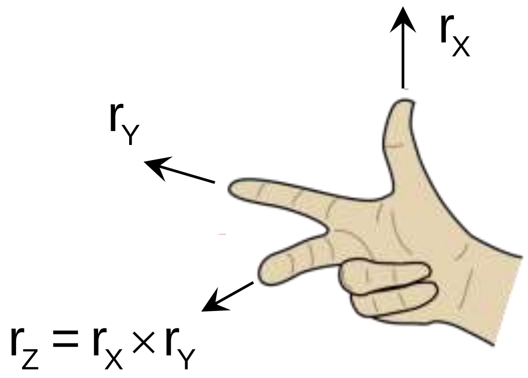
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^C R_W X, \quad \text{rows: } r_X,\ r_Y,\ r_Z$$

$${}^C R_W \in SO(3)$$

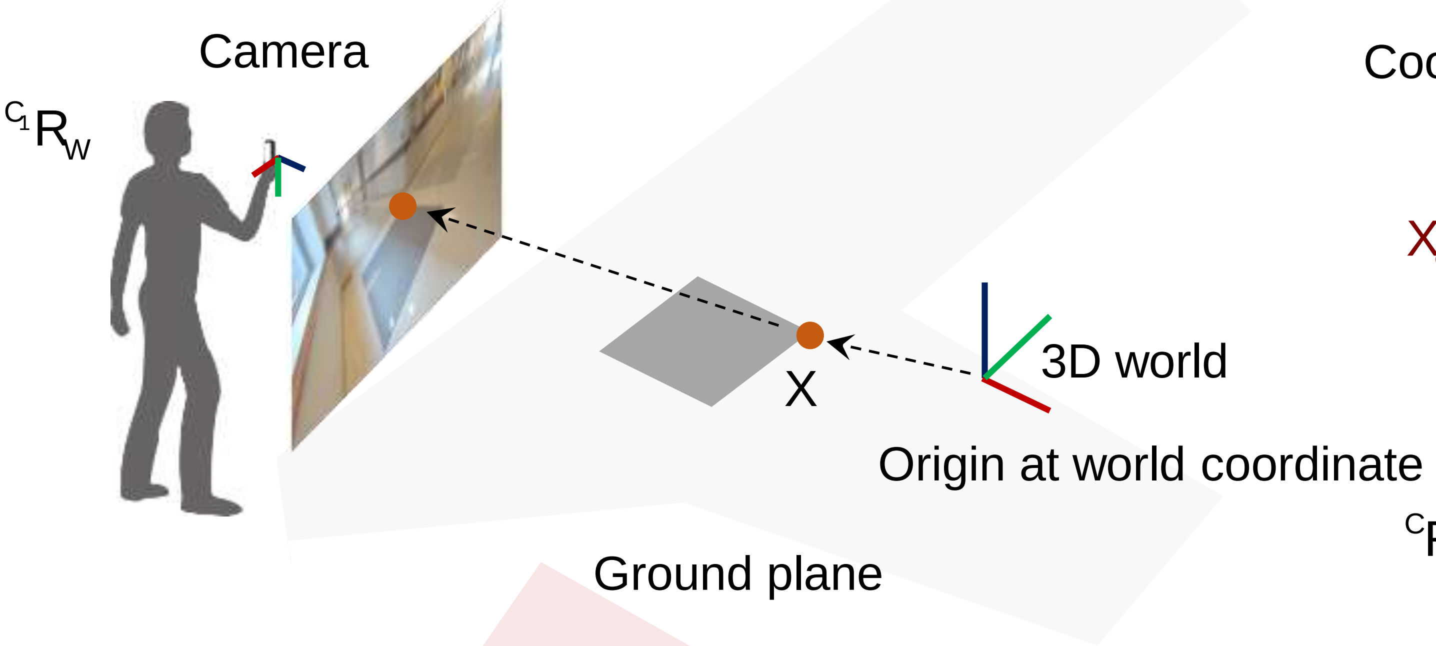
- Orthogonal matrix → $\left({}^C R_W\right)^T \left({}^C R_W\right) = I_3, \quad \det\left({}^C R_W\right) = 1$
- Right hand rule



r_Y : camera y axis seen from the world coord.

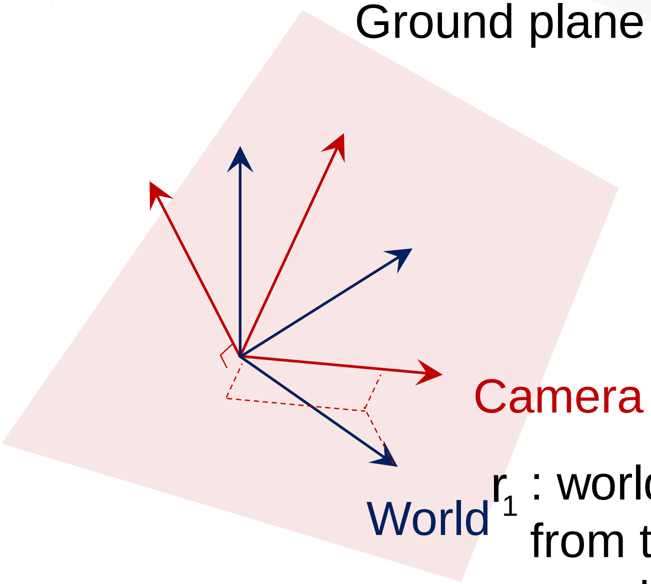


Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

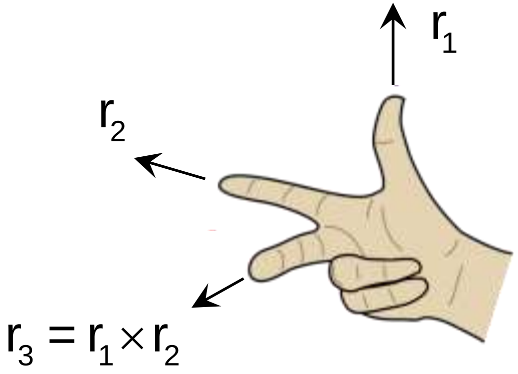
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^C R_W X$$



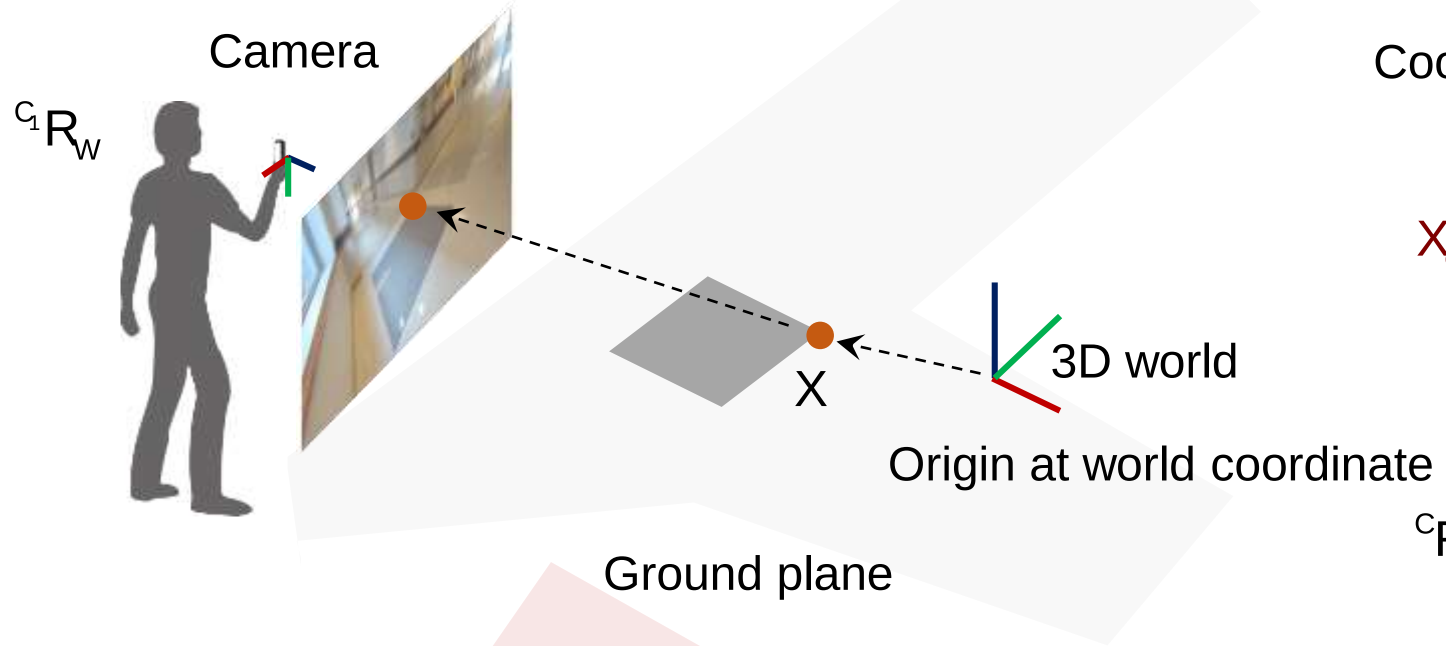
r_1 : world x axis seen from the camera coord.

$${}^C R_W \in SO(3)$$

- Orthogonal matrix → $\left({}^C R_W\right)^T \left({}^C R_W\right) = I_3, \quad \det\left({}^C R_W\right) = 1$
- Right hand rule



Coordinate Transform (Rotation)

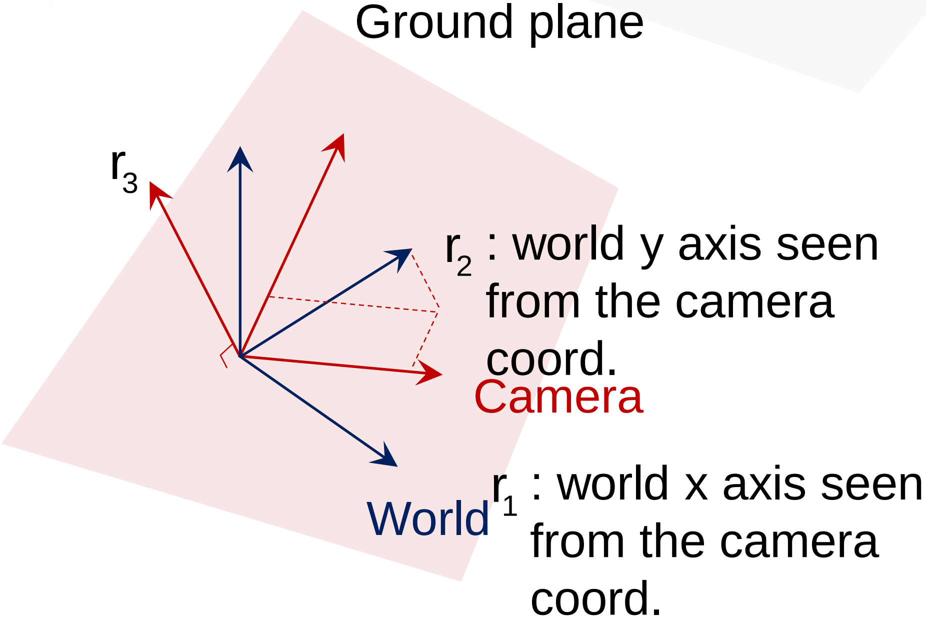


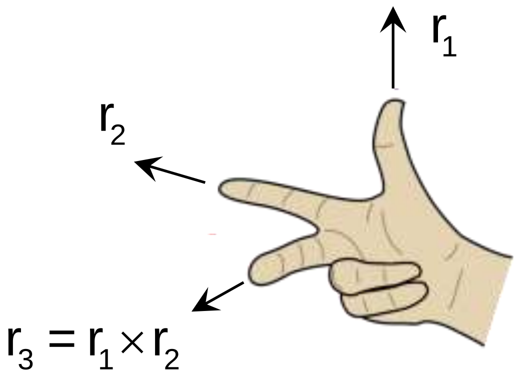
Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X, \quad \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}$$

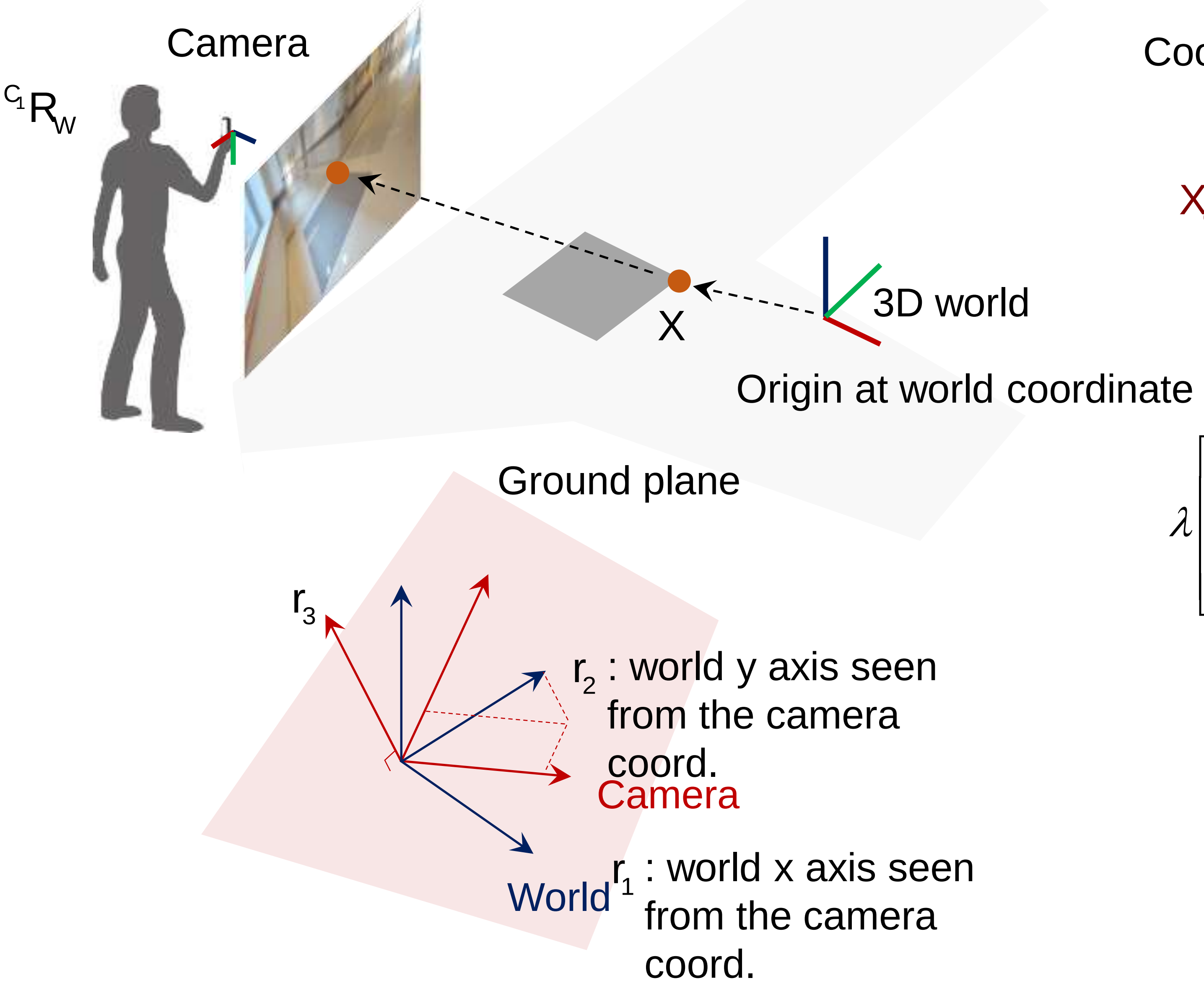
$^{C}R_W \in SO(3)$

- Orthogonal matrix → $\left({}^{C}R_W\right)^T \left({}^{C}R_W\right) = I_3, \quad \det\left({}^{C}R_W\right) = 1$
- Right hand rule





Camera Projection (Pure Rotation)



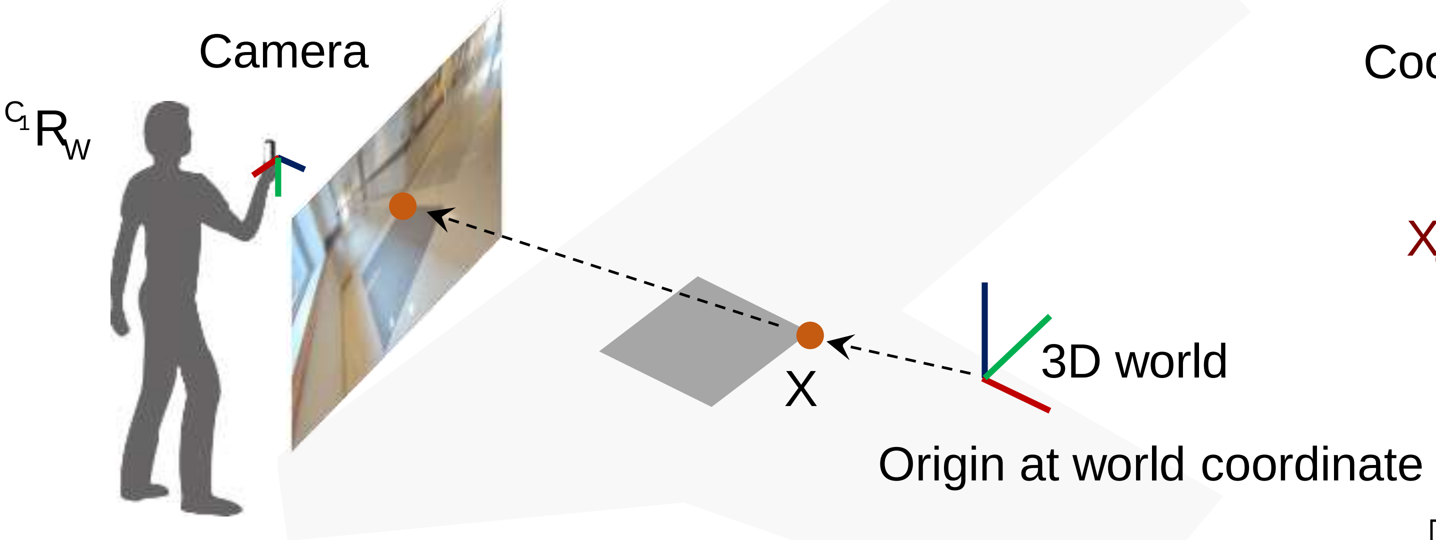
Coordinate transformation from world to camera:

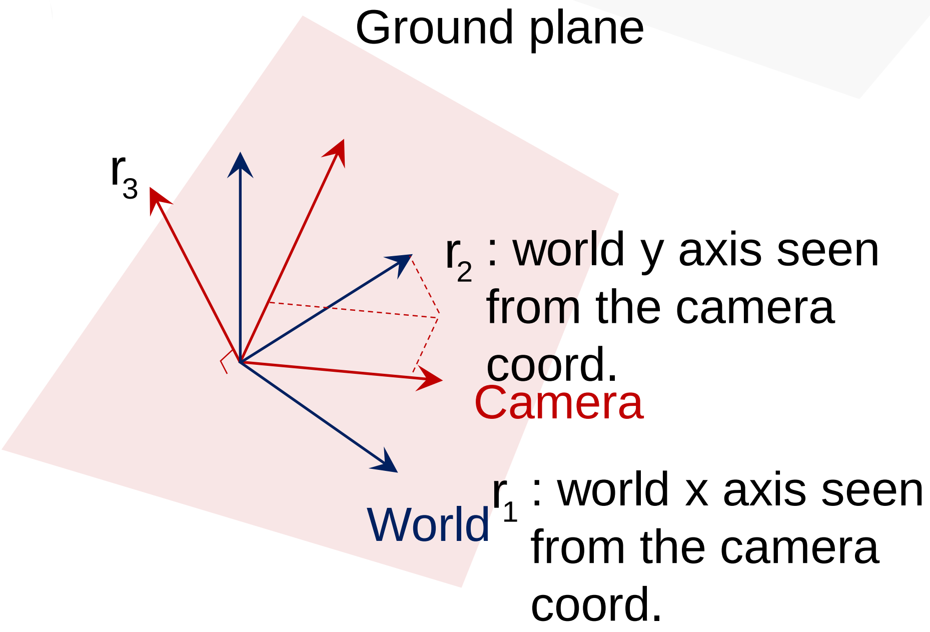
$$X_C = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^{C}R_W X$$

Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

Camera Projection (Pure Rotation)





Coordinate transformation from world to camera:

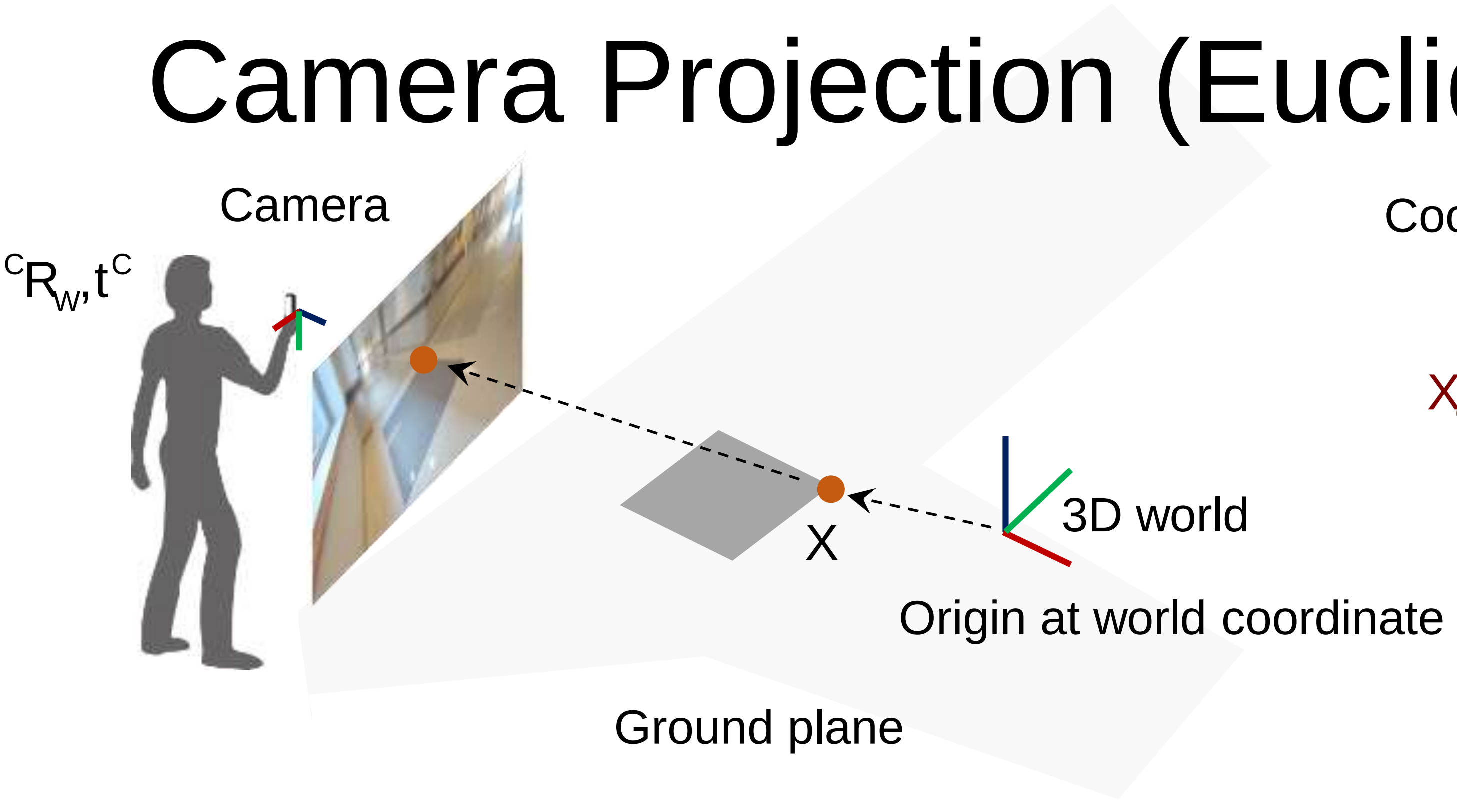
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^{C}R_{W} X$$

Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}, \quad K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

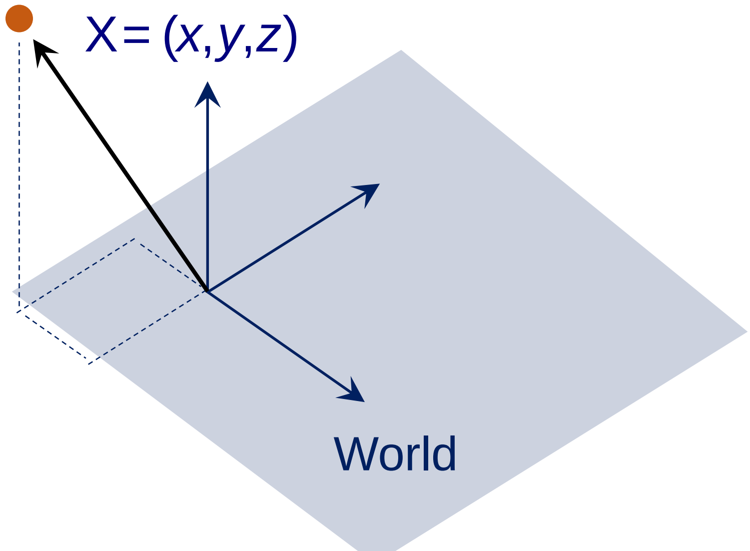
$$= K \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Camera Projection (Euclidean Transform)

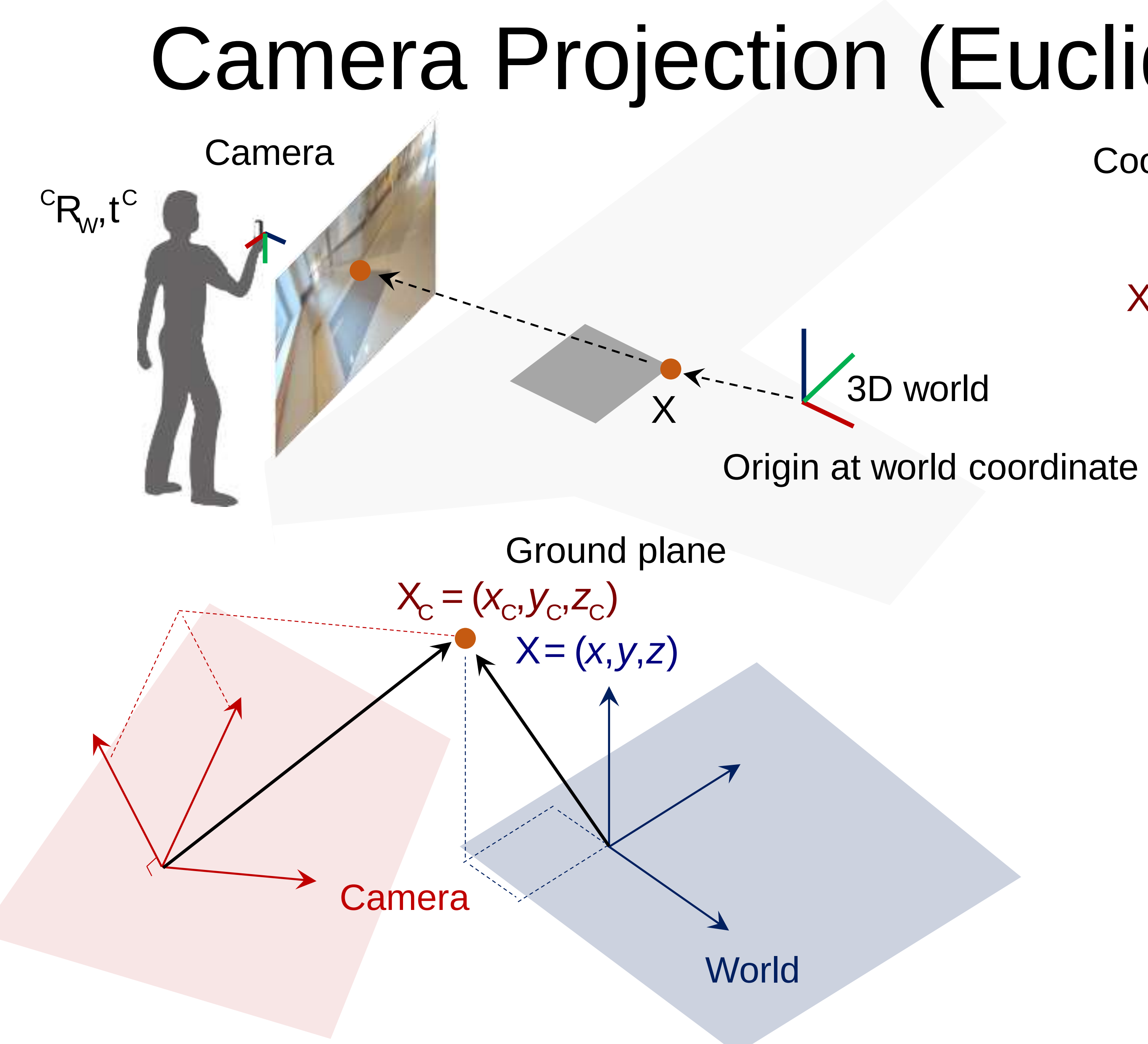


Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_{W} X$$



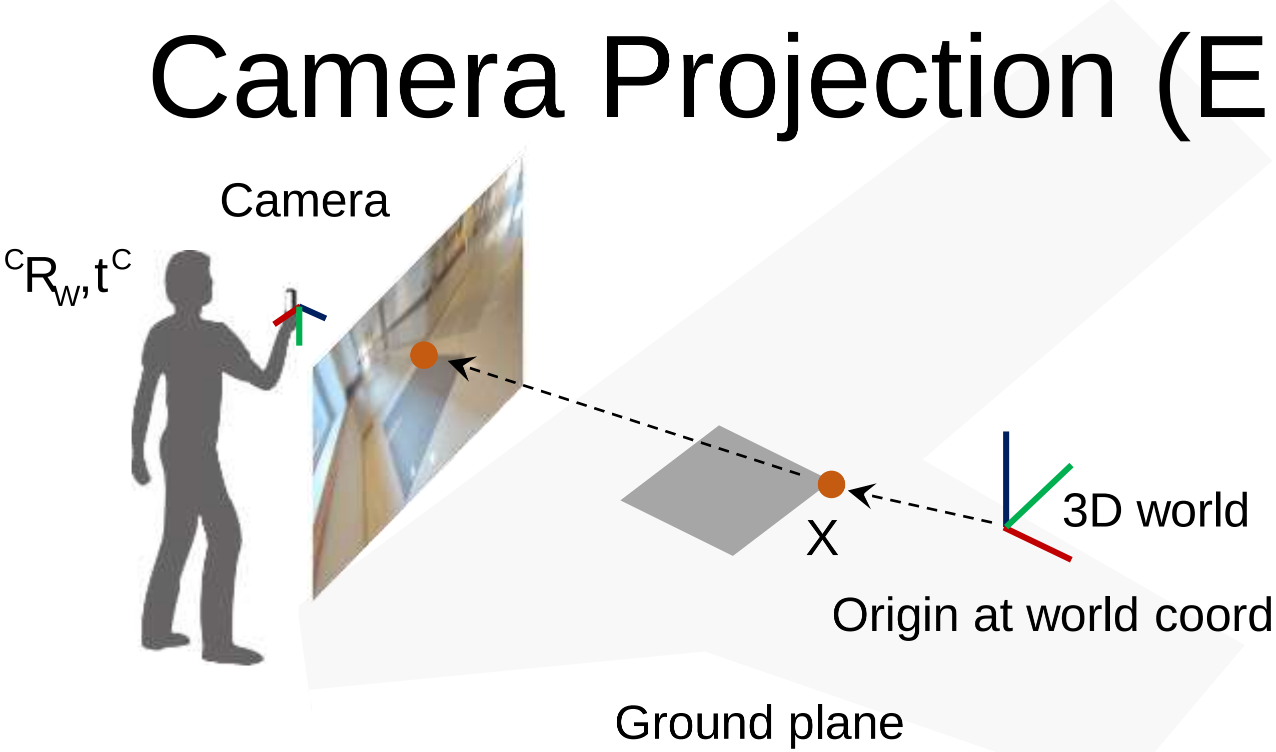
Camera Projection (Euclidean Transform)



Coordinate transformation from world to camera:

$$X_C = {}^{C}R_{W}X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

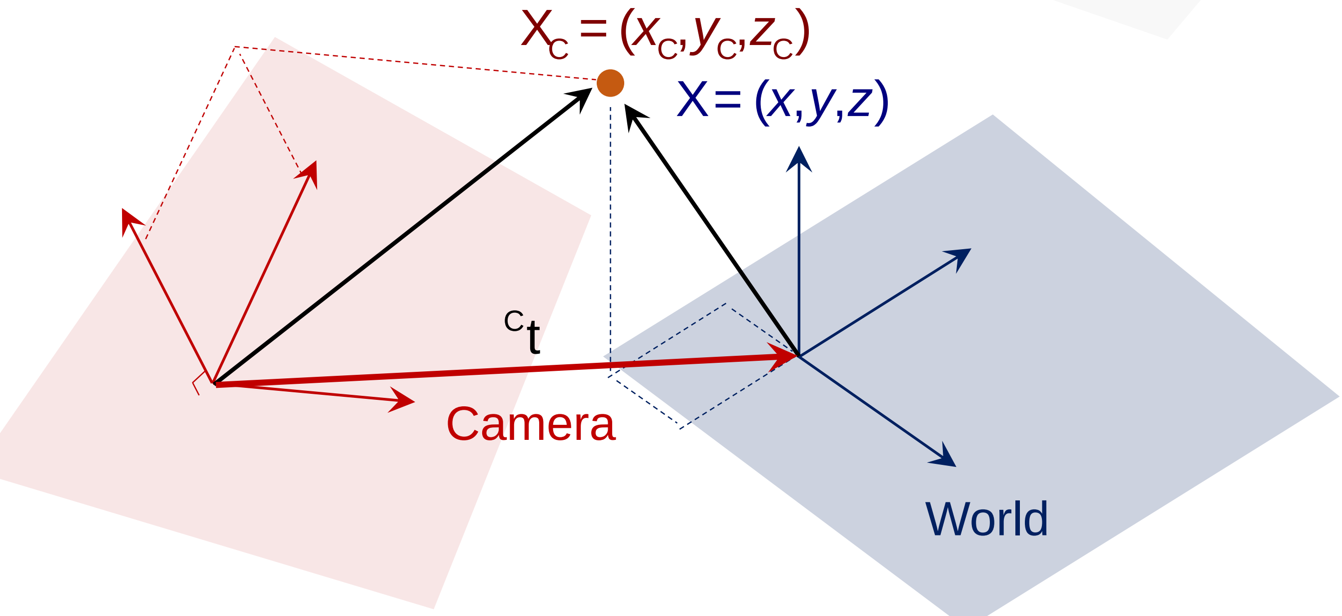
Camera Projection (Euclidean Transform)



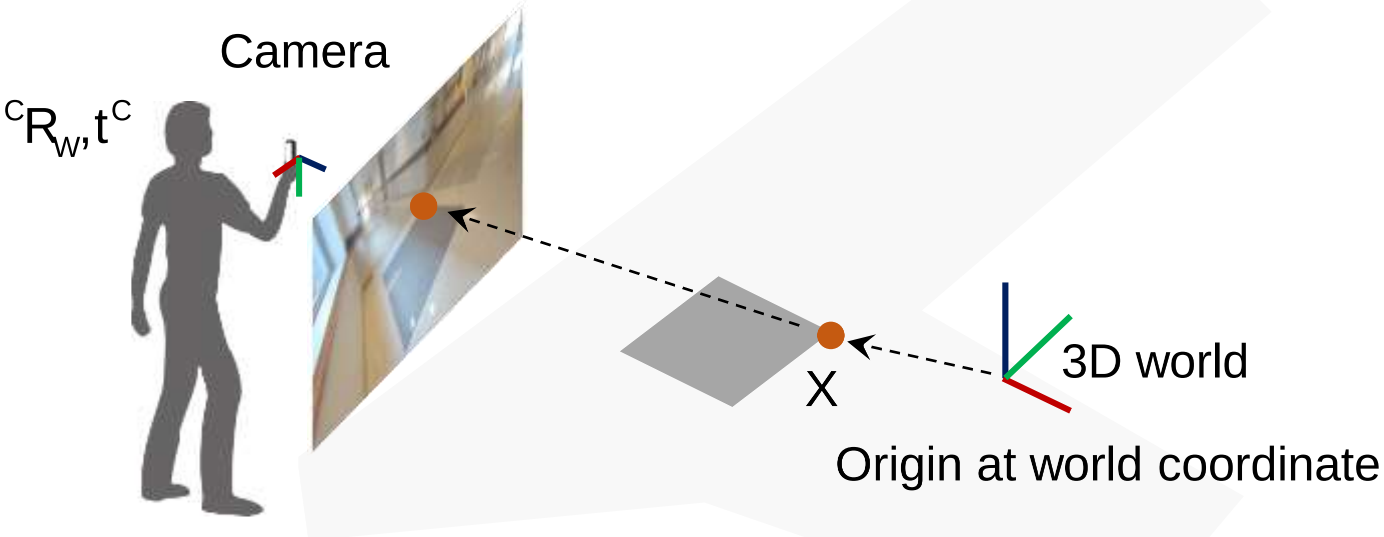
Coordinate transformation from world to camera:

$$X_C = {}^{C}R_{W}X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

Where ${}^{C}t$ is translation from world to camera seen from camera.



Geometric Interpretation

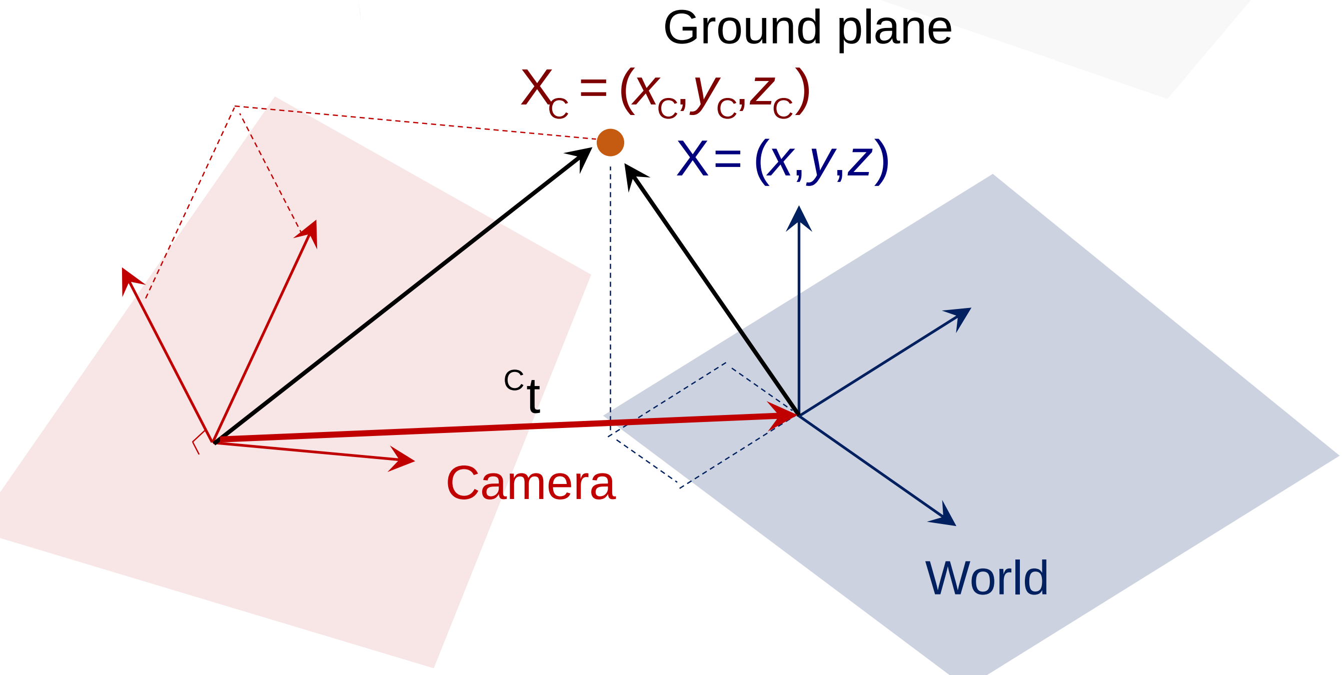


Coordinate transformation from world to camera:

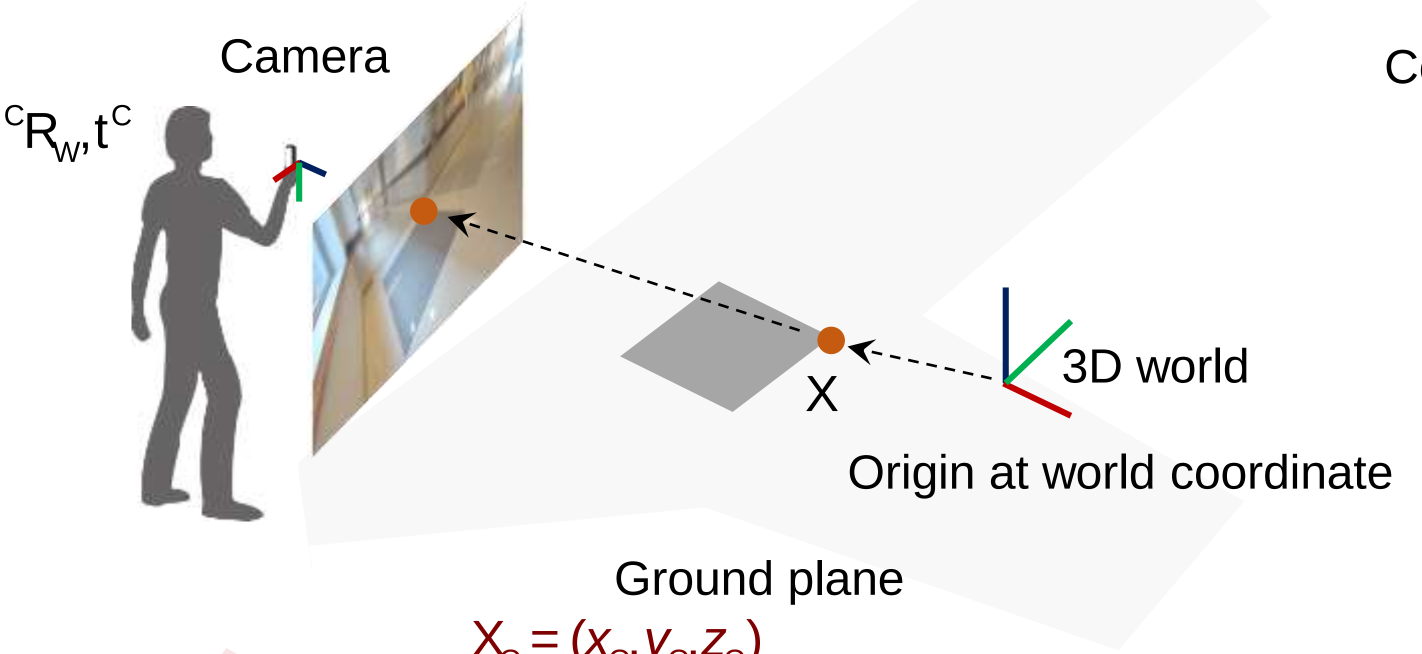
$$X_C = {}^{C}R_W X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

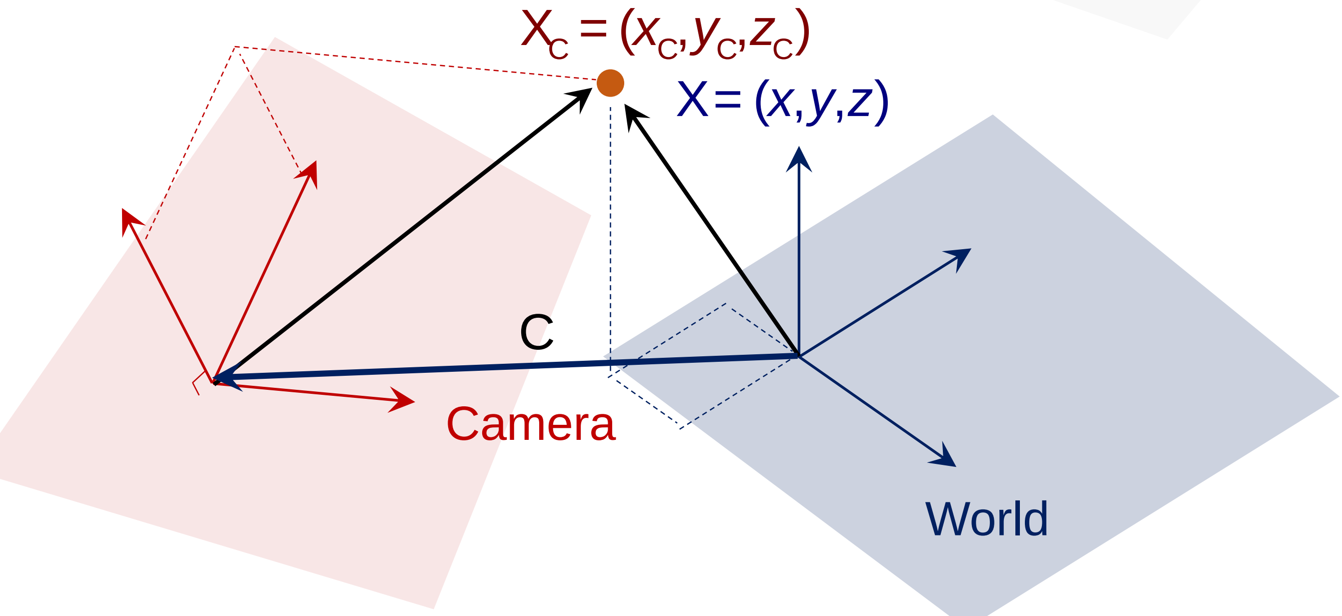
where ^{C}t is translation from world to camera seen from camera.

Rotate and then, translate.



Geometric Interpretation





Coordinate transformation from world to camera:

$$X_C = {}^{C}R_{W} X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

where ${}^{C}t$ is translation from world to camera seen from camera.

Rotate and then, translate.

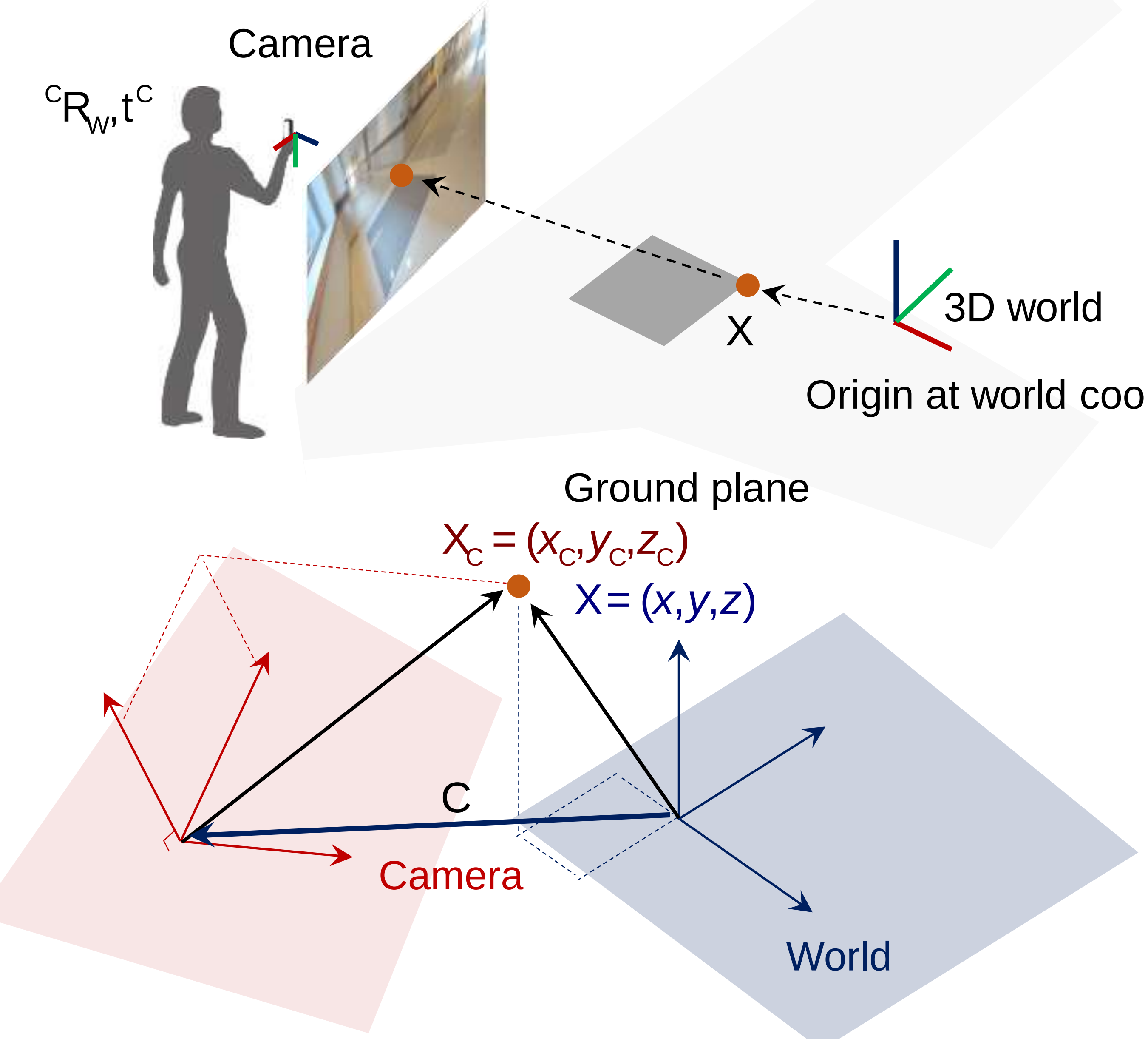
---

cf) Translate and then, rotate.

$$X_C = {}^{C}R_{W}(X - C) = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} 1 & & & -C_x \\ & 1 & & -C_y \\ & & 1 & -C_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

where C is translation from world to camera seen from world.

Camera Projection Matrix



Coordinate transformation from world to camera:

$$X_C = {}^C R_W X + {}^C t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \left[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{{}^C R_W} \; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^C t} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Image Projection: Sanity Check

$$C = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

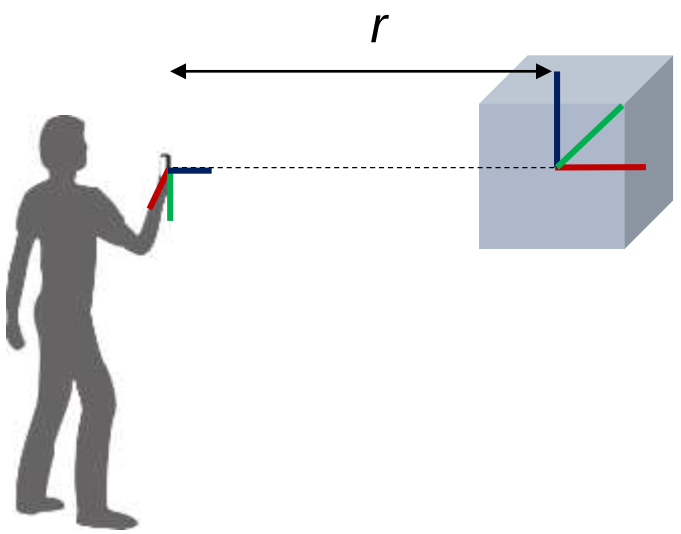


Image Projection

$$C=\begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad R=\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

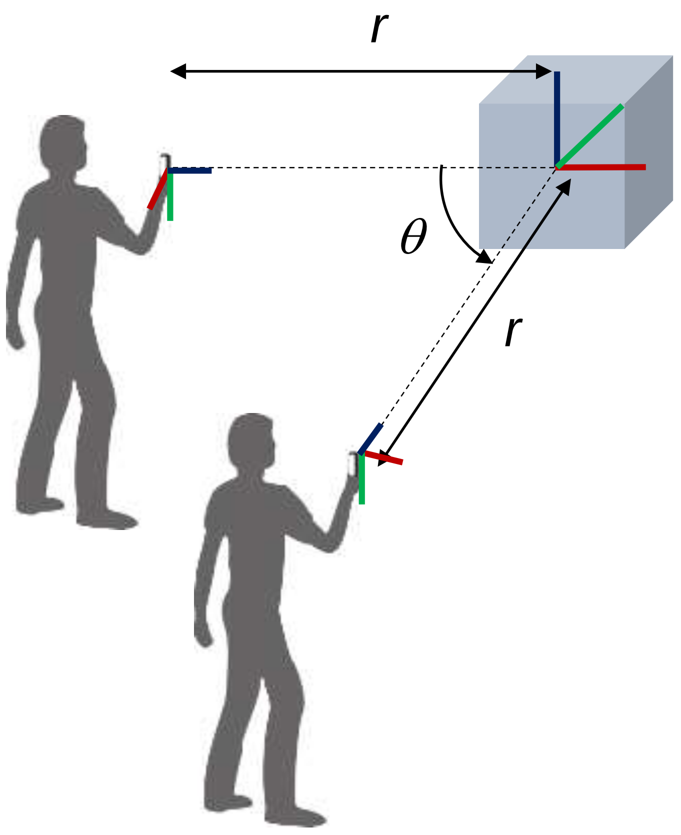
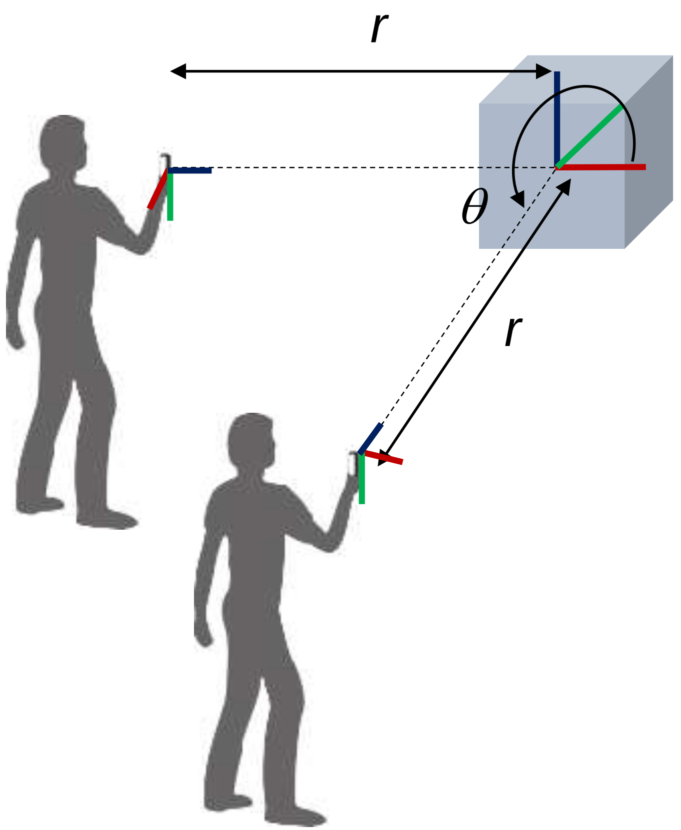


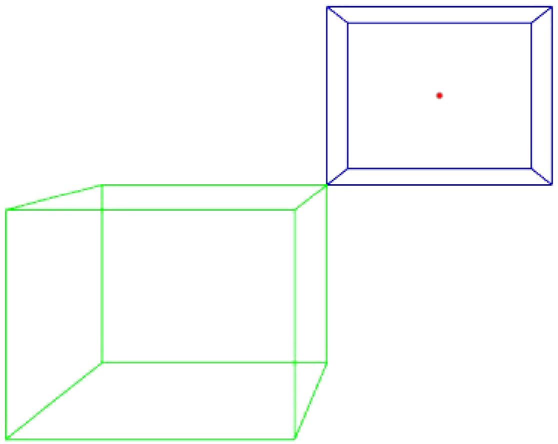
Image Projection



$$C = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

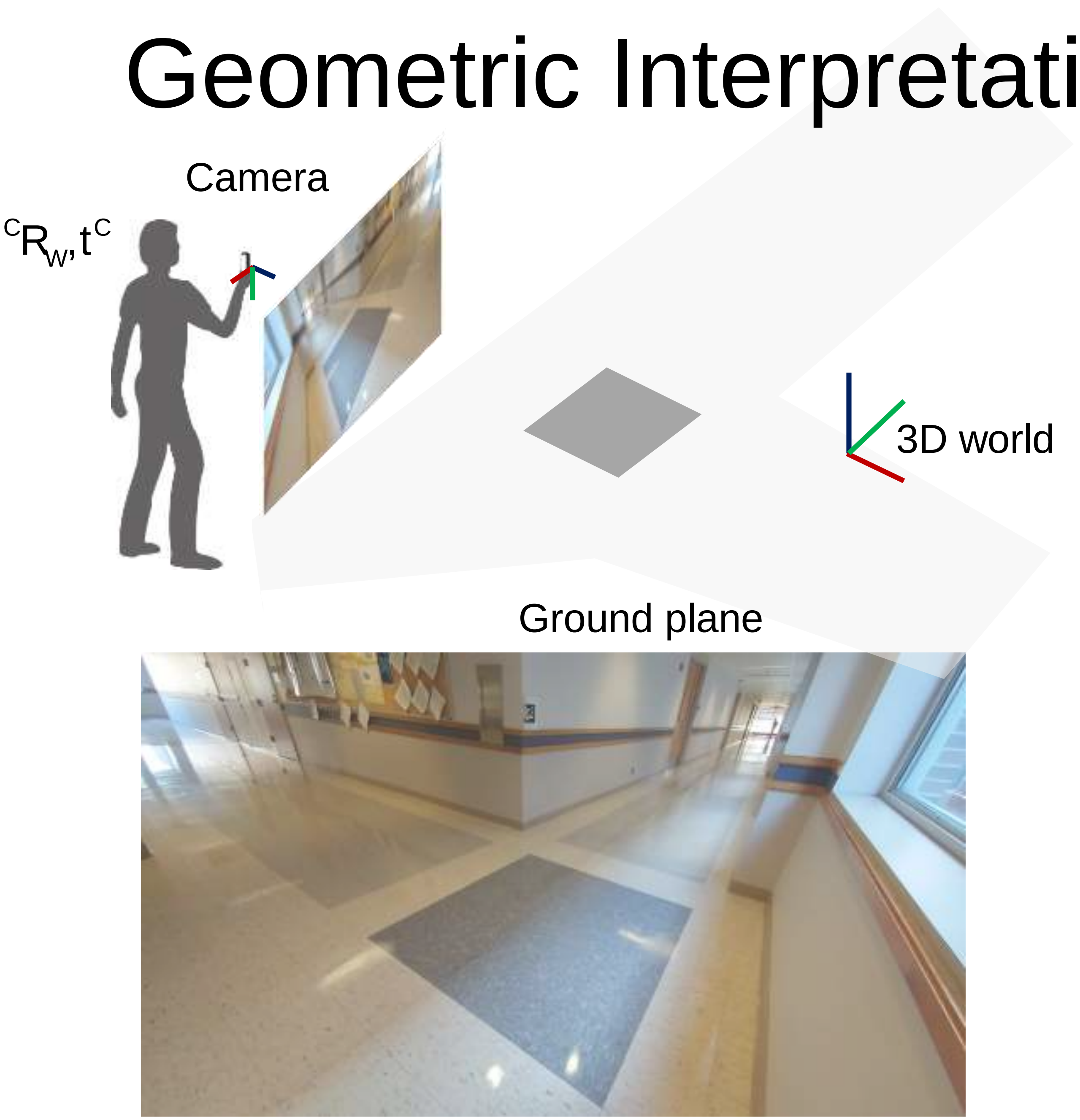
$$C = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \\ -\cos\theta & -\sin\theta & 0 \end{bmatrix}$$

Image Projection



```
K = [200 0 100;
     0 200 100;
     0 0 1];

radius = 5;

theta = 0:0.02:2*pi;

for i = 1 : length(theta)
    camera_offset = [radius*cos(theta(i)); radius*sin(theta(i)); 0];
    camera_center = camera_offset + center_of_mass';

    rz = camera_center-center_of_mass';
    rz = rz / norm(rz);
    ry = [0 0 1]';
    rx = Vec2Skew(ry)*rz; % cross product
    R = [rx'; ry'; rz'];
    C = camera_center;
    P = K * R * [ eye(3) -C];

    proj = [];
    for j = 1 : size(sqaure_point,1)
        u = P * [sqaure_point(j,:)';1];
        proj(j,:) = u'/u(3);
    end
end
```

Geometric Interpretation



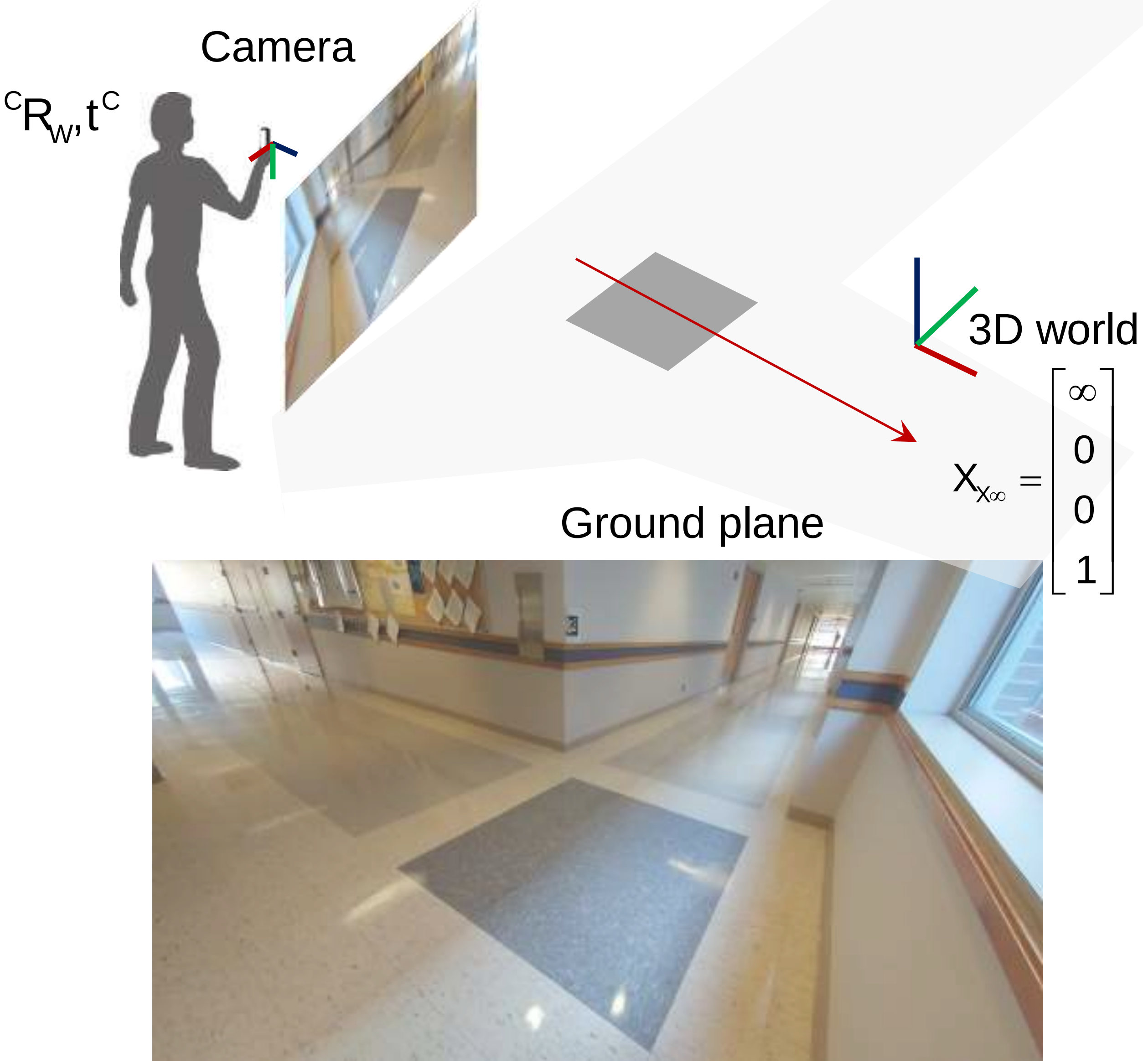
Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \Big[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} \;\; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \Big] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\rightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What does each number mean?

Geometric Interpretation



$$X_{X\infty} = \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Camera projection of world point:

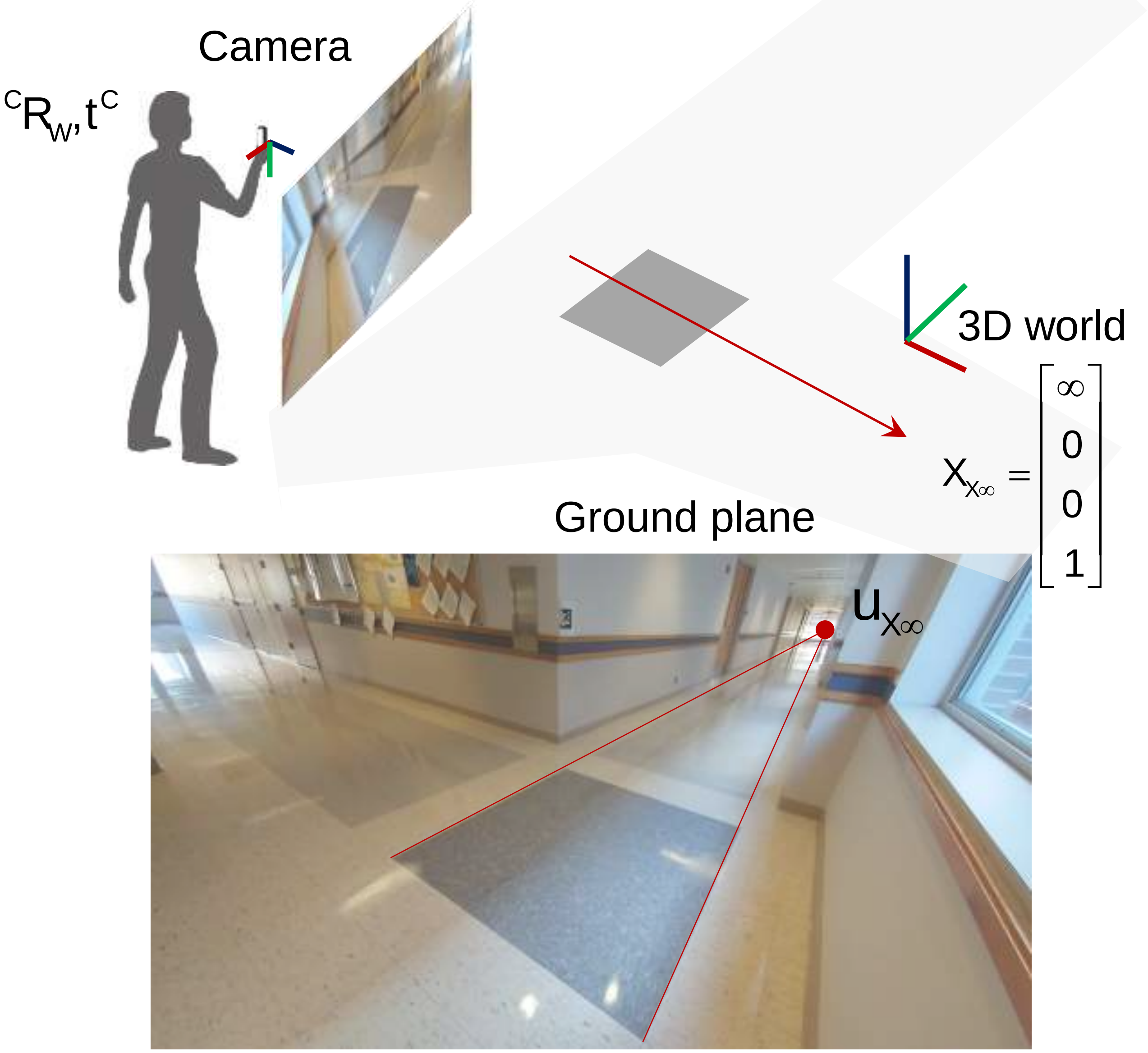
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \left[{}^{C}R_{W} \;\; {}^{C}t \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}, \quad {}^{C}R_{W} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix}, \quad {}^{C}t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

What is point at infinity in world x direction?

Geometric Interpretation



$$X_{X\infty} = \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} & \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

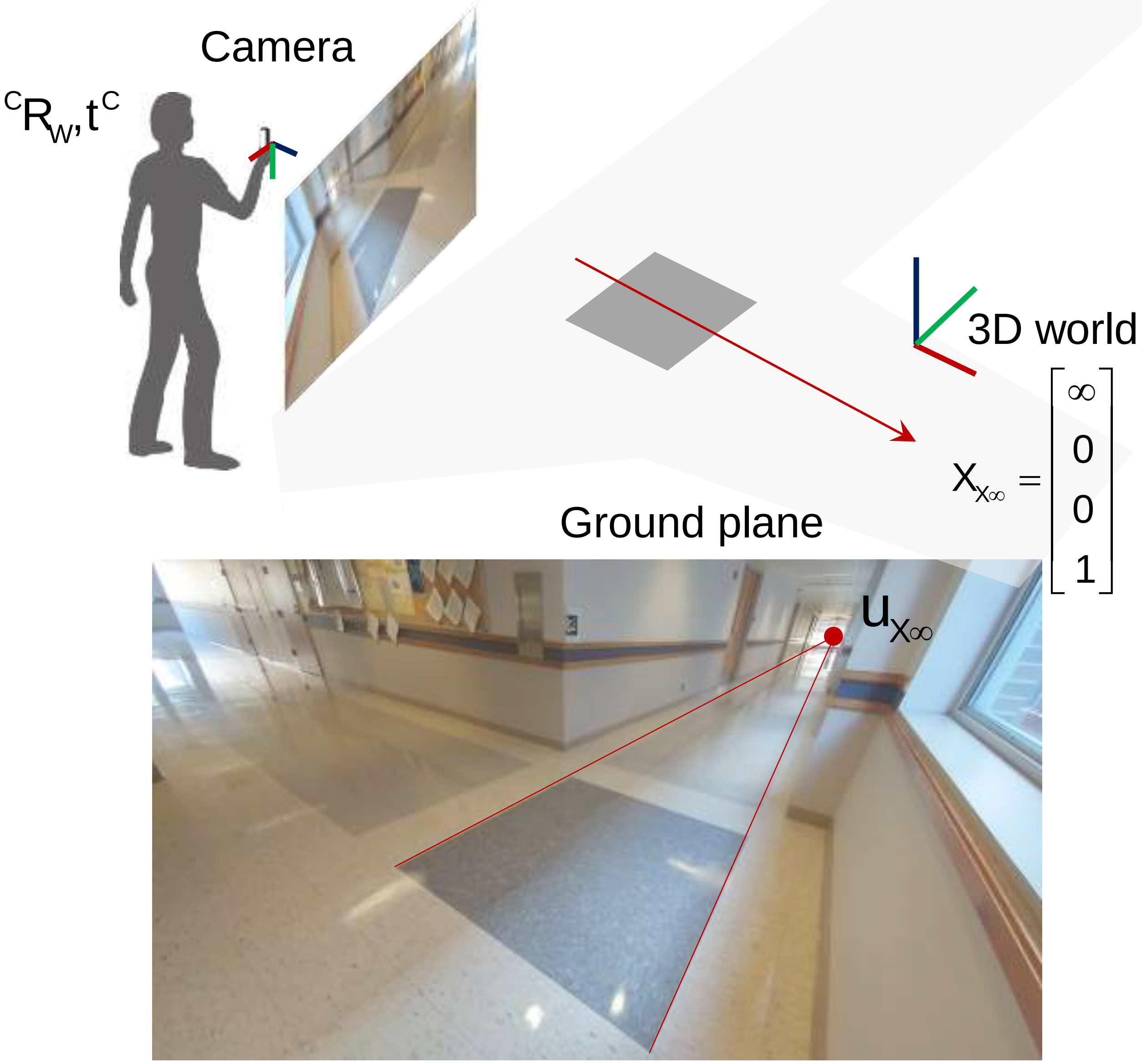
$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

What is point at infinity in world x direction?

This point is at infinite but finite in image.

Point at infinity

Geometric Interpretation



Camera projection of world point:

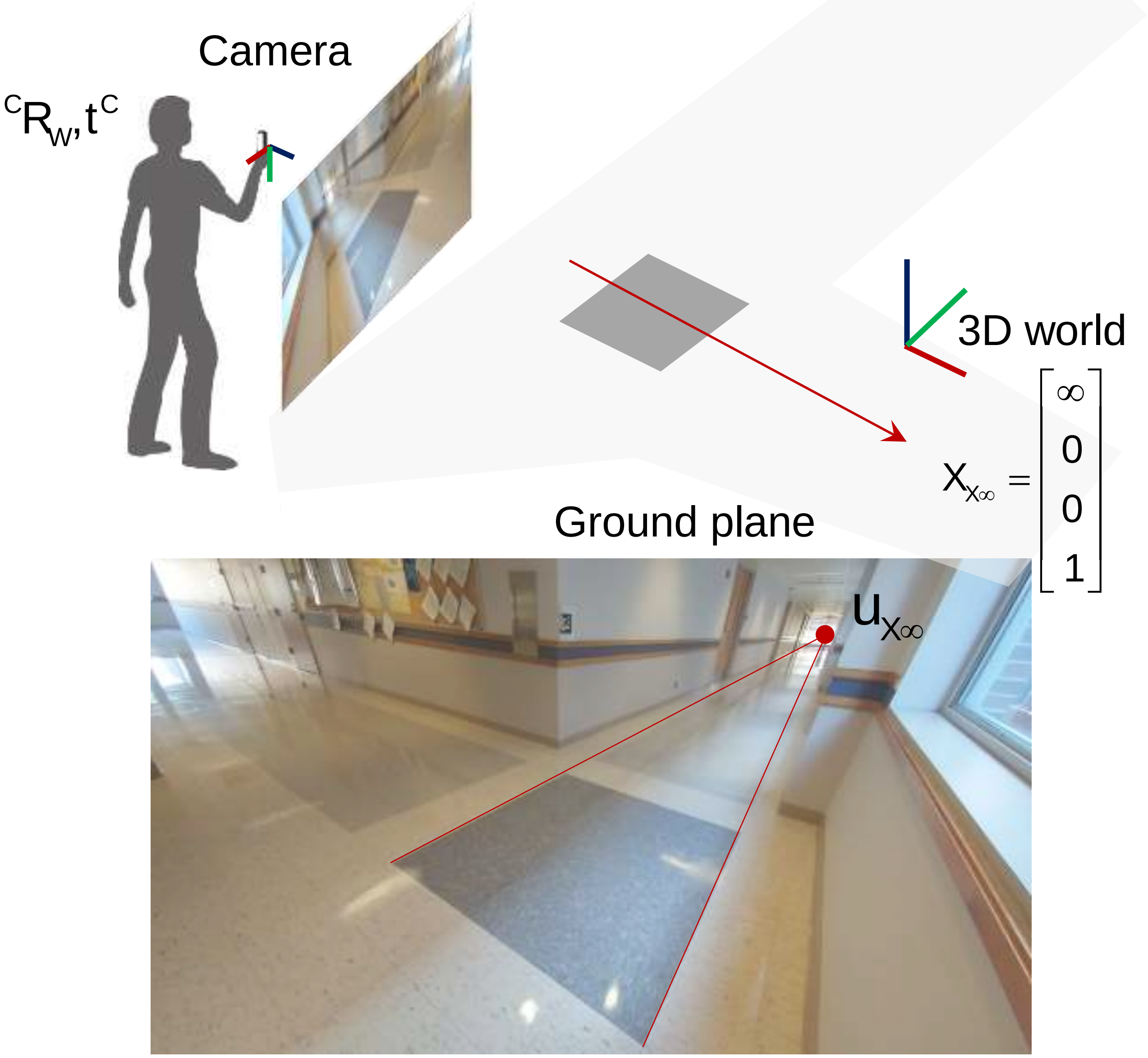
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} & \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

What is point at infinity in world x direction?

This point is at infinite but finite in image.

Geometric Interpretation



Camera projection of world point:

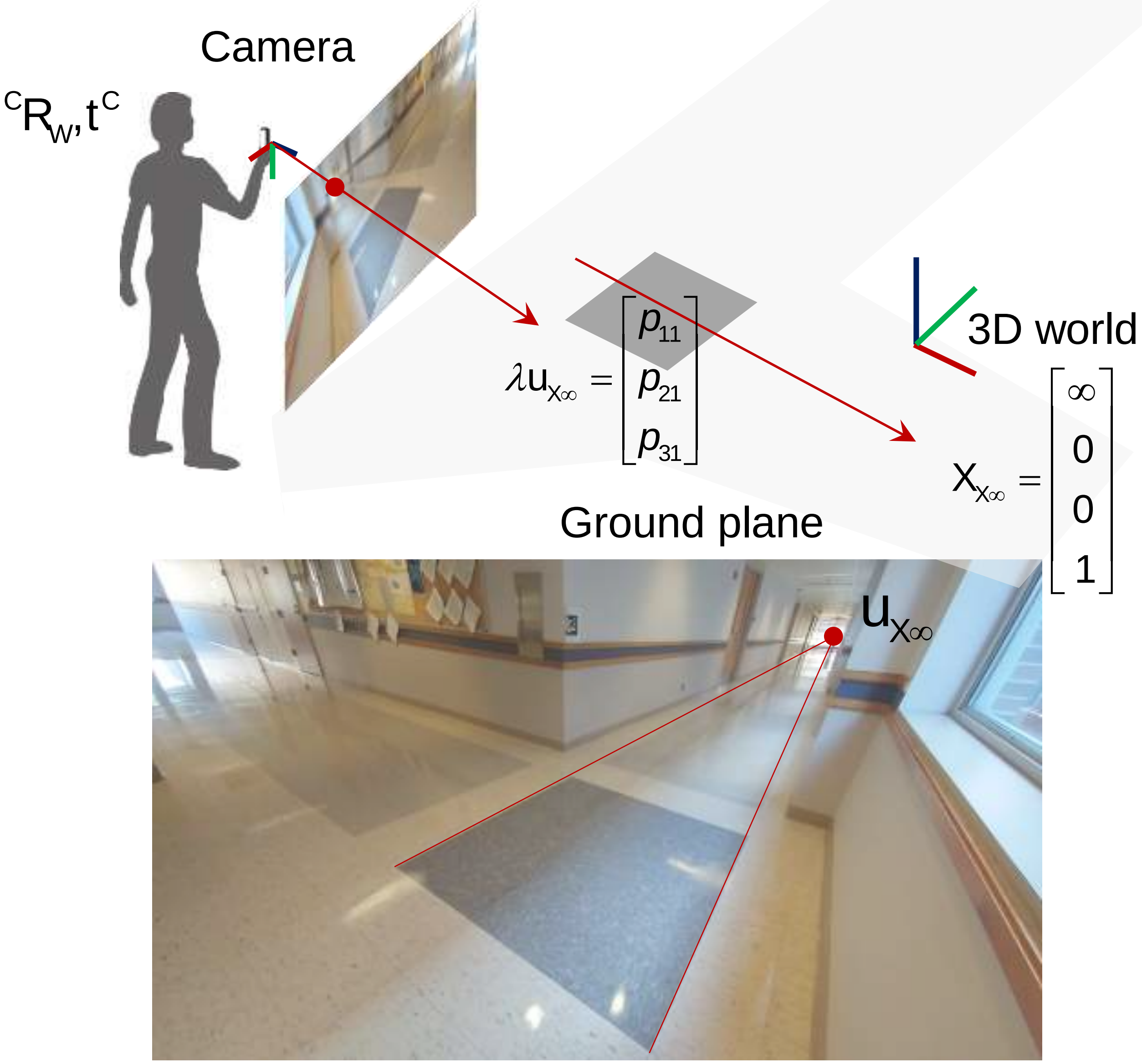
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \Bigg[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} \; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \Bigg] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad u = \lim_{X \to \infty} \frac{p_{11}X + p_{14}}{p_{31}X + p_{34}} = \frac{p_{11}}{p_{31}}$$

$$v = \lim_{X \to \infty} \frac{p_{21}X + p_{24}}{p_{31}X + p_{34}} = \frac{p_{21}}{p_{31}}$$

Geometric Interpretation



Camera projection of world point:

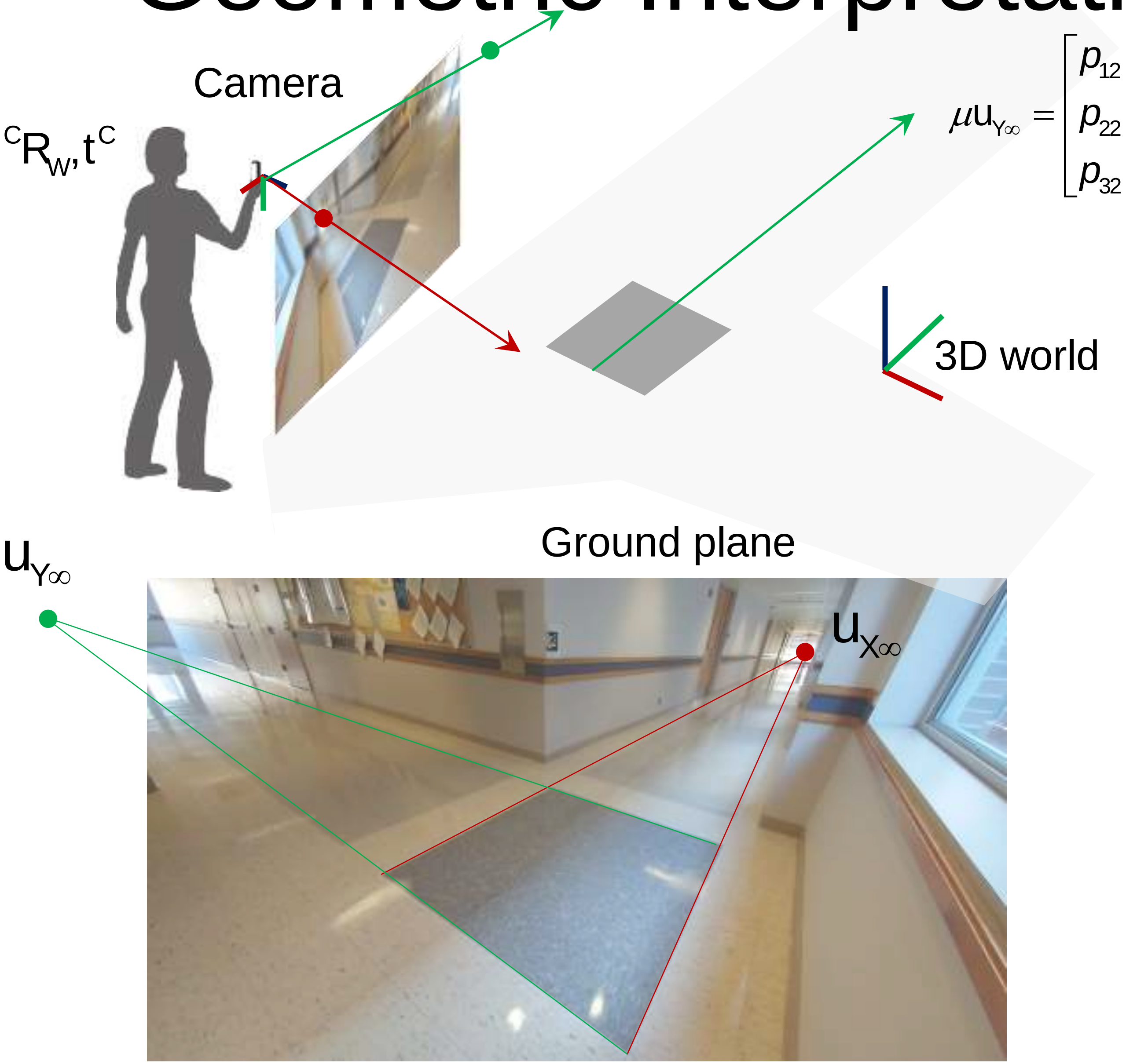
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} & \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\rightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\rightarrow \quad u = \lim_{X \to \infty} \frac{p_{11}X + p_{14}}{p_{31}X + p_{34}} = \frac{p_{11}}{p_{31}} \qquad v = \lim_{X \to \infty} \frac{p_{21}X + p_{24}}{p_{31}X + p_{34}} = \frac{p_{21}}{p_{31}}$$

$$\rightarrow \lambda u_{X\infty} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix}$$

Geometric Interpretation



$$\mu u_{Y\infty} = \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix}$$

Camera projection of world point:

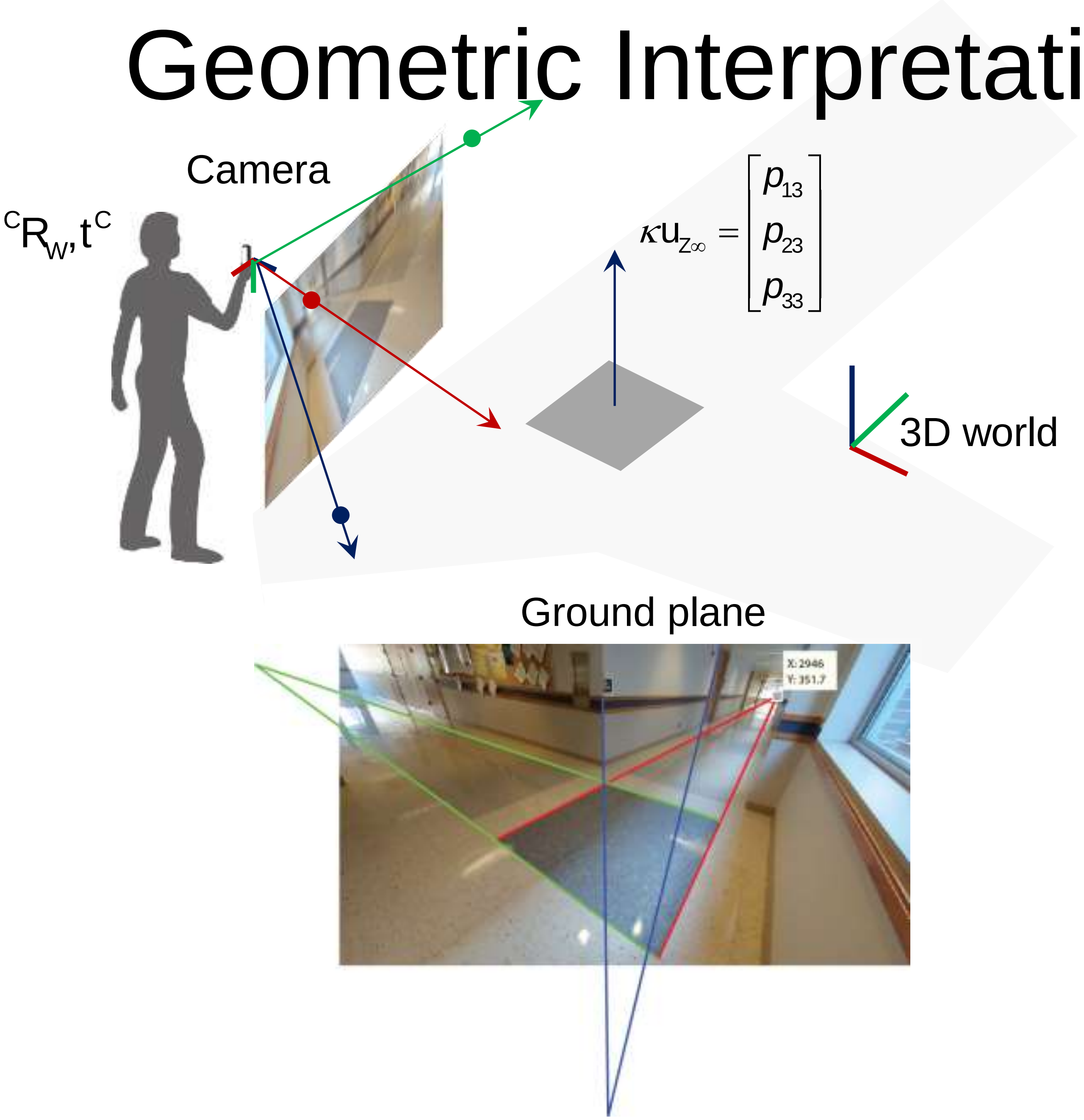
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} & \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} 0 \\ \infty \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad u = \lim_{X \to \infty} \frac{p_{12}Y + p_{14}}{p_{32}Y + p_{34}} = \frac{p_{12}}{p_{32}} \qquad v = \lim_{X \to \infty} \frac{p_{22}Y + p_{24}}{p_{32}Y + p_{34}} = \frac{p_{22}}{p_{32}}$$

$$\longrightarrow \mu u_{Y\infty} = \mu \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix}$$

Geometric Interpretation



Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

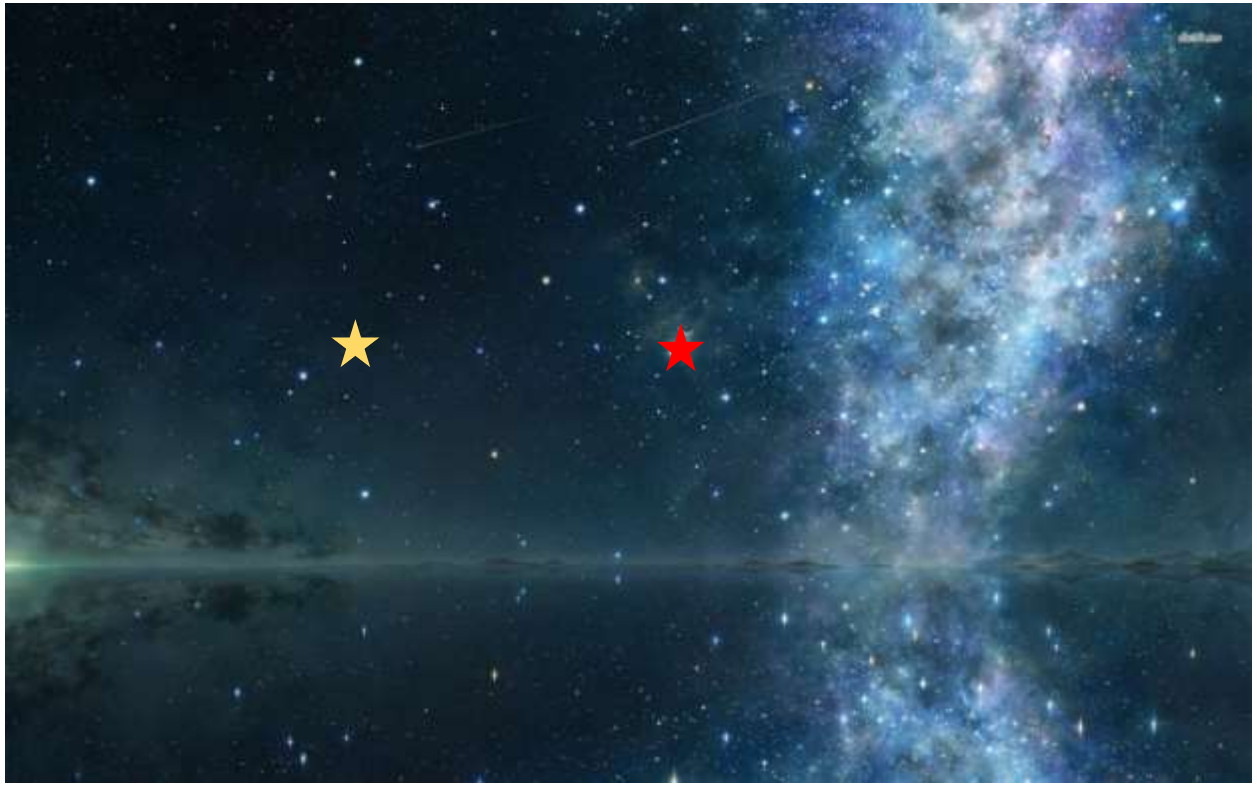
(K, $^{C}R_{W}$, ^{C}t)

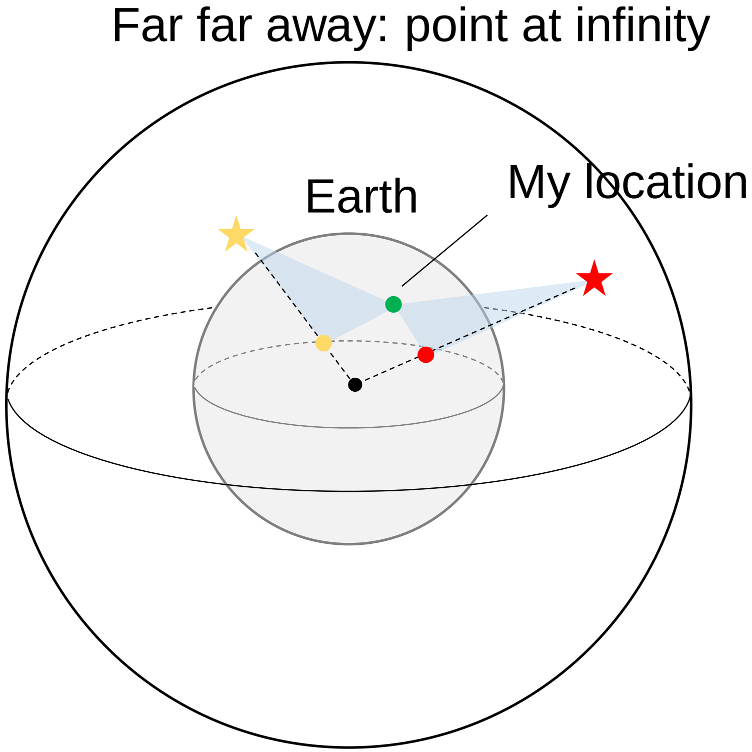
$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \infty \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad u = \lim_{X \to \infty} \frac{p_{13}Z + p_{14}}{p_{33}Z + p_{34}} = \frac{p_{13}}{p_{33}} \qquad v = \lim_{X \to \infty} \frac{p_{23}Z + p_{24}}{p_{33}Z + p_{34}} = \frac{p_{23}}{p_{33}}$$

$$\longrightarrow \kappa u_{Z\infty} = \kappa \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix}$$

Celestial Navigation



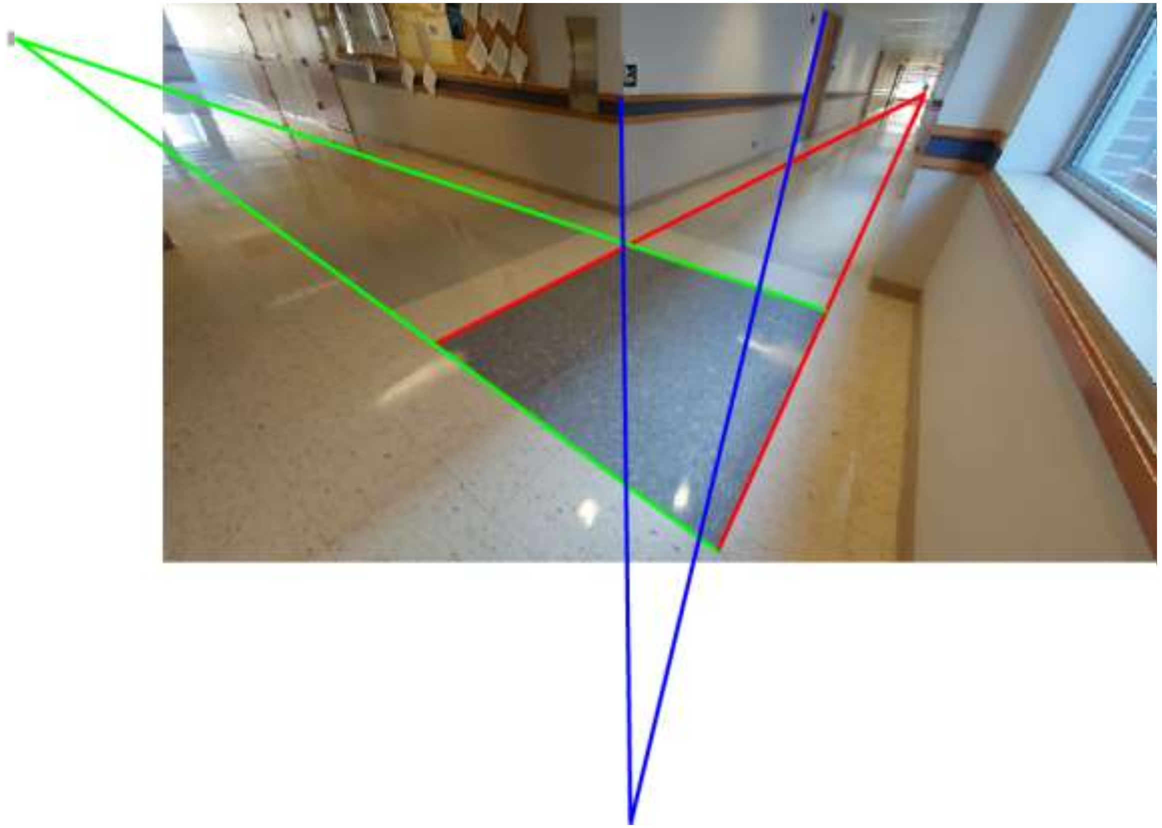


Practice

$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\text{pix}$$

$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

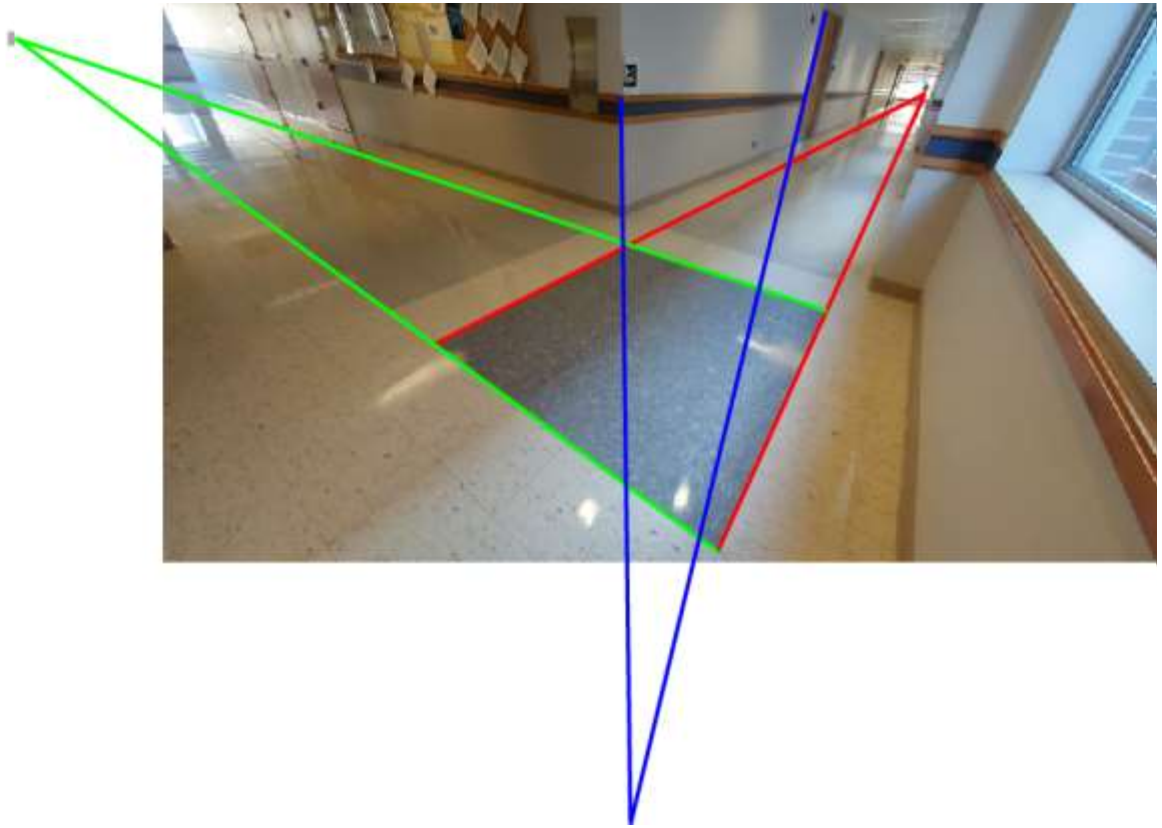


Practice

$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\text{pix}$$

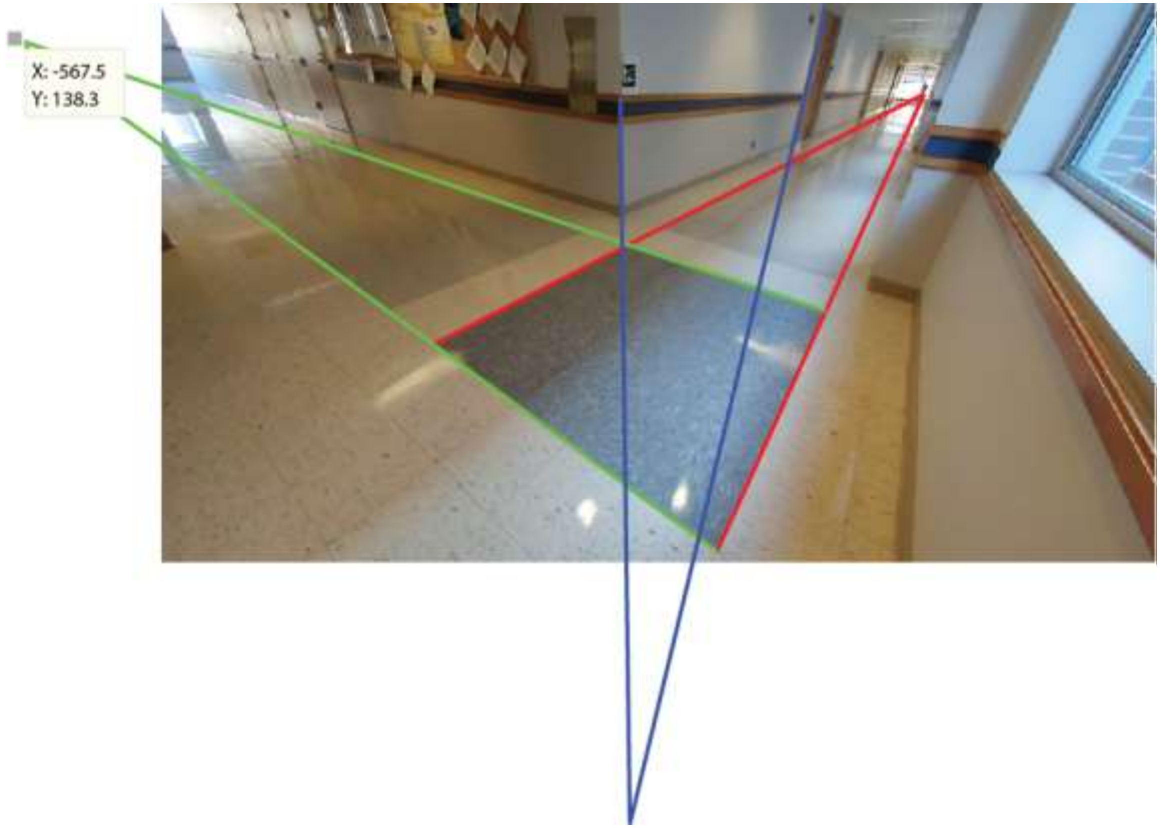
$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$



$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} -0.2374 & -0.9565 & 2.0138 & 1.2723 \\ 0.0578 & -1.5763 & 0.2404 & 1.2508 \\ 0.0004 & -0.0005 & 0.0007 & 0.0006 \end{bmatrix}$$

[-0.2374 0.0578]/ 0.0004
=
[-593.5   144.5]



$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\text{pix}$$

$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} -0.2374 & -0.9565 & 2.0138 & 1.2723 \\ 0.0578 & -1.5763 & 0.2404 & 1.2508 \\ 0.0004 & -0.0005 & 0.0007 & 0.0006 \end{bmatrix}$$

```
R = [-0.8496    0.0498    0.5731
  -0.3216   -0.8203   -0.4067
   0.4180   -0.5299    0.6835];

C = [0.0070
   0.7520
  -0.2738];

P = K * R * [eye(3) -C]
u_x = P(1:2,1)/P(3,1)
u_y = P(1:2,2)/P(3,2)
u_z = P(1:2,3)/P(3,3)
```

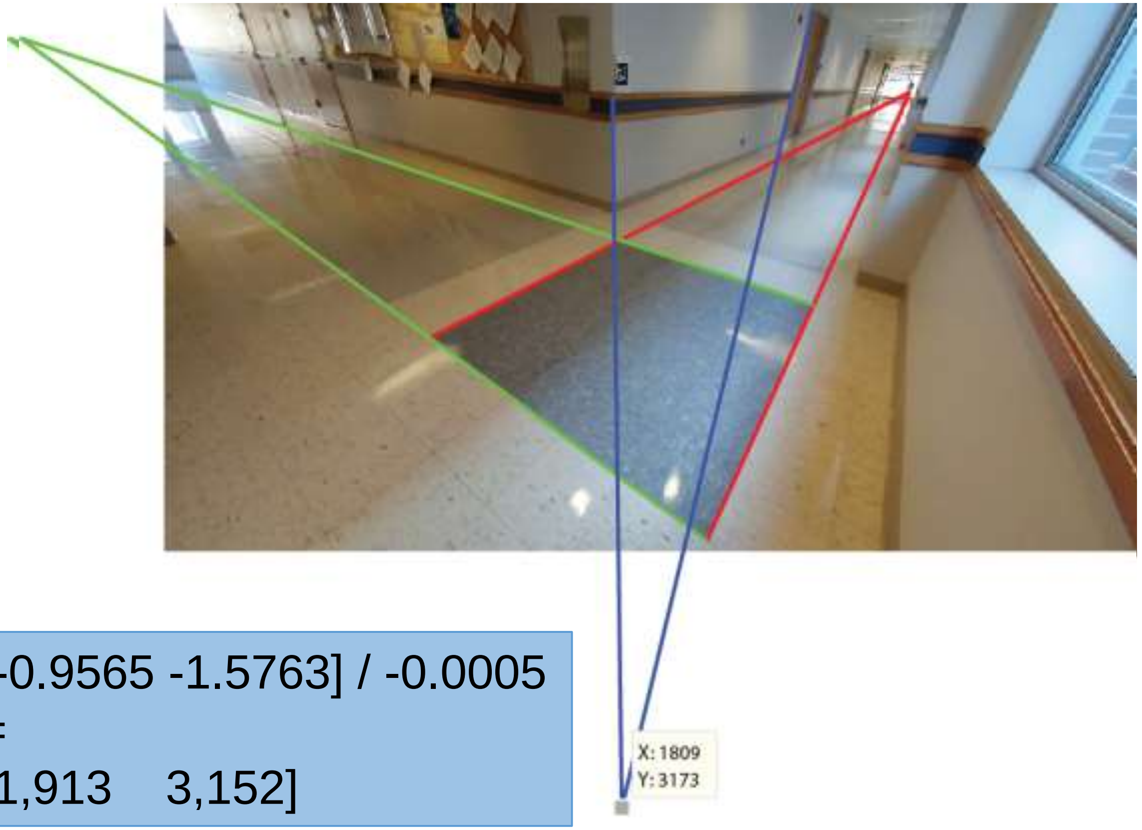
Practice

$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\text{pix}$$

$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

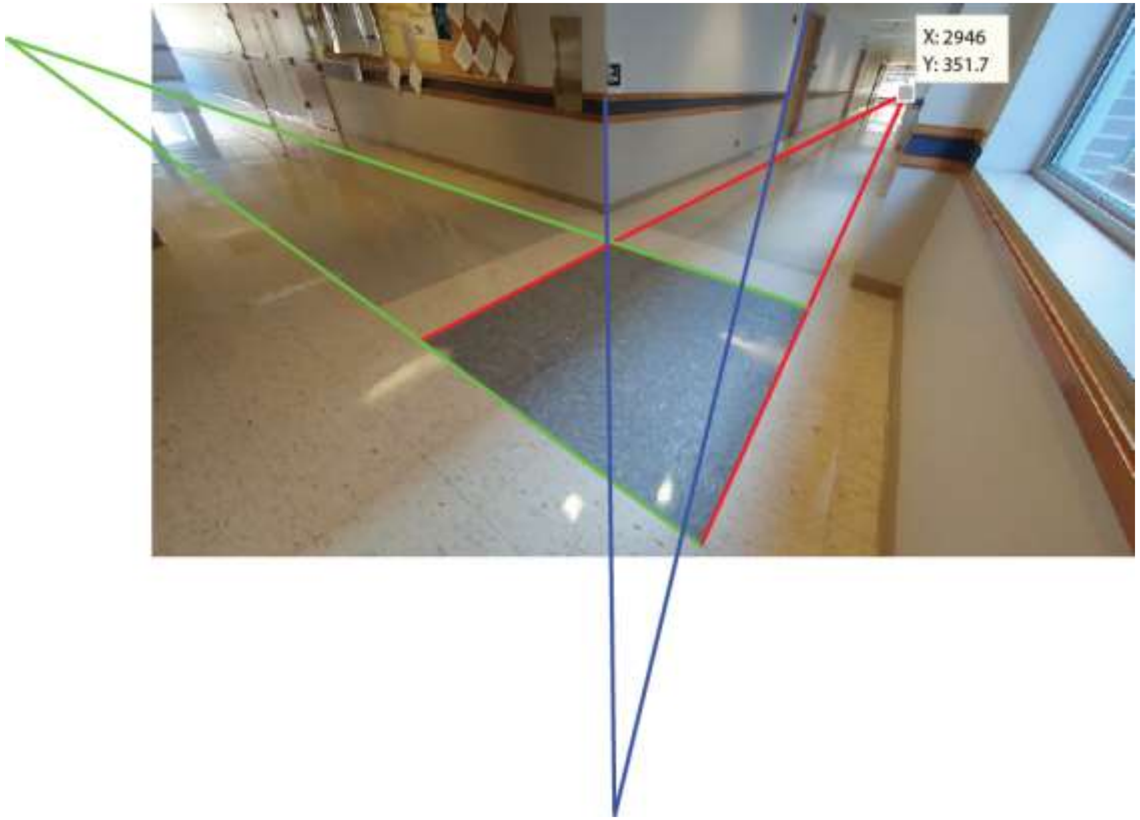
$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} -0.2374 & -0.9565 & 2.0138 & 1.2723 \\ 0.0578 & -1.5763 & 0.2404 & 1.2508 \\ 0.0004 & -0.0005 & 0.0007 & 0.0006 \end{bmatrix}$$



[-0.9565 -1.5763] / -0.0005
=
[1,913 3,152]

```
R = [-0.8496   0.0498   0.5731
  -0.3216  -0.8203  -0.4067
   0.4180  -0.5299   0.6835];

C = [0.0070
   0.7520
  -0.2738];

P = K * R * [eye(3) -C]
u_x = P(1:2,1)/P(3,1)
u_y = P(1:2,2)/P(3,2)
u_z = P(1:2,3)/P(3,3)
```

[2.0138 0.2404]/0.0007
=
[2877   343]



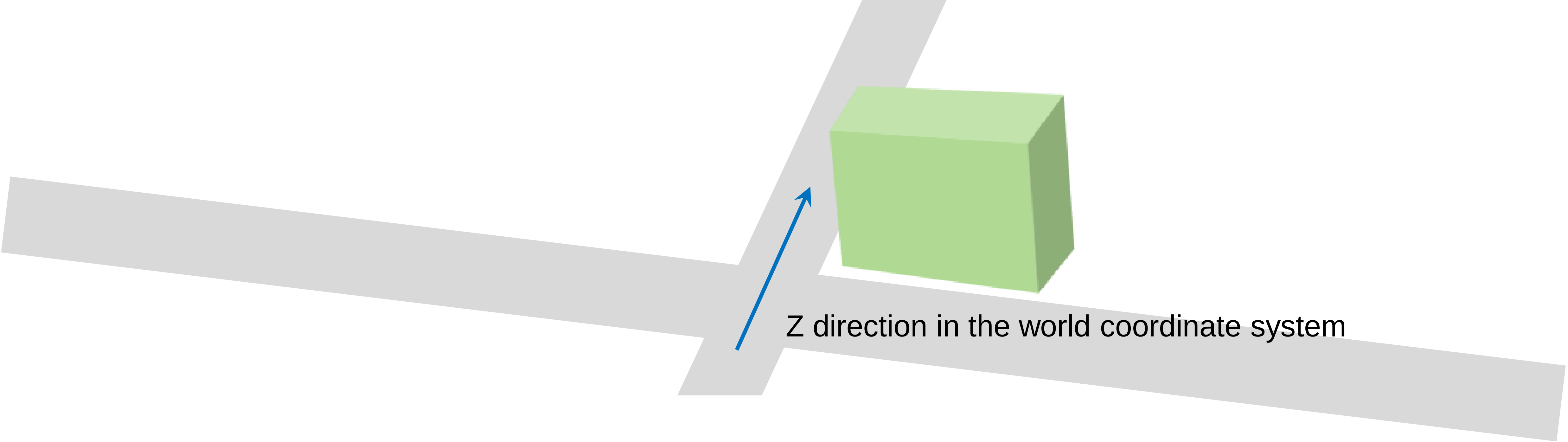
$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\text{pix}$$

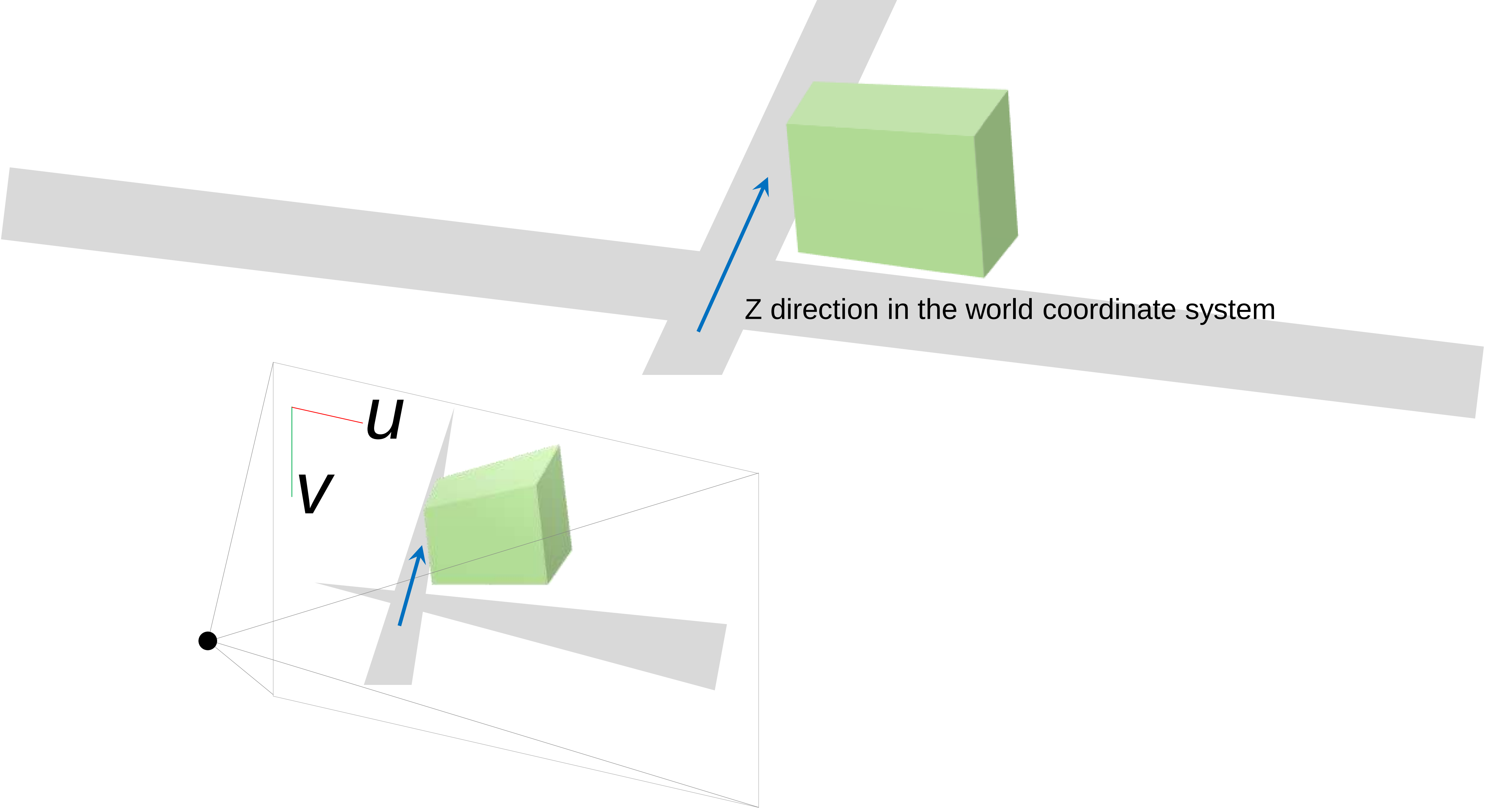
$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\text{pix}$$

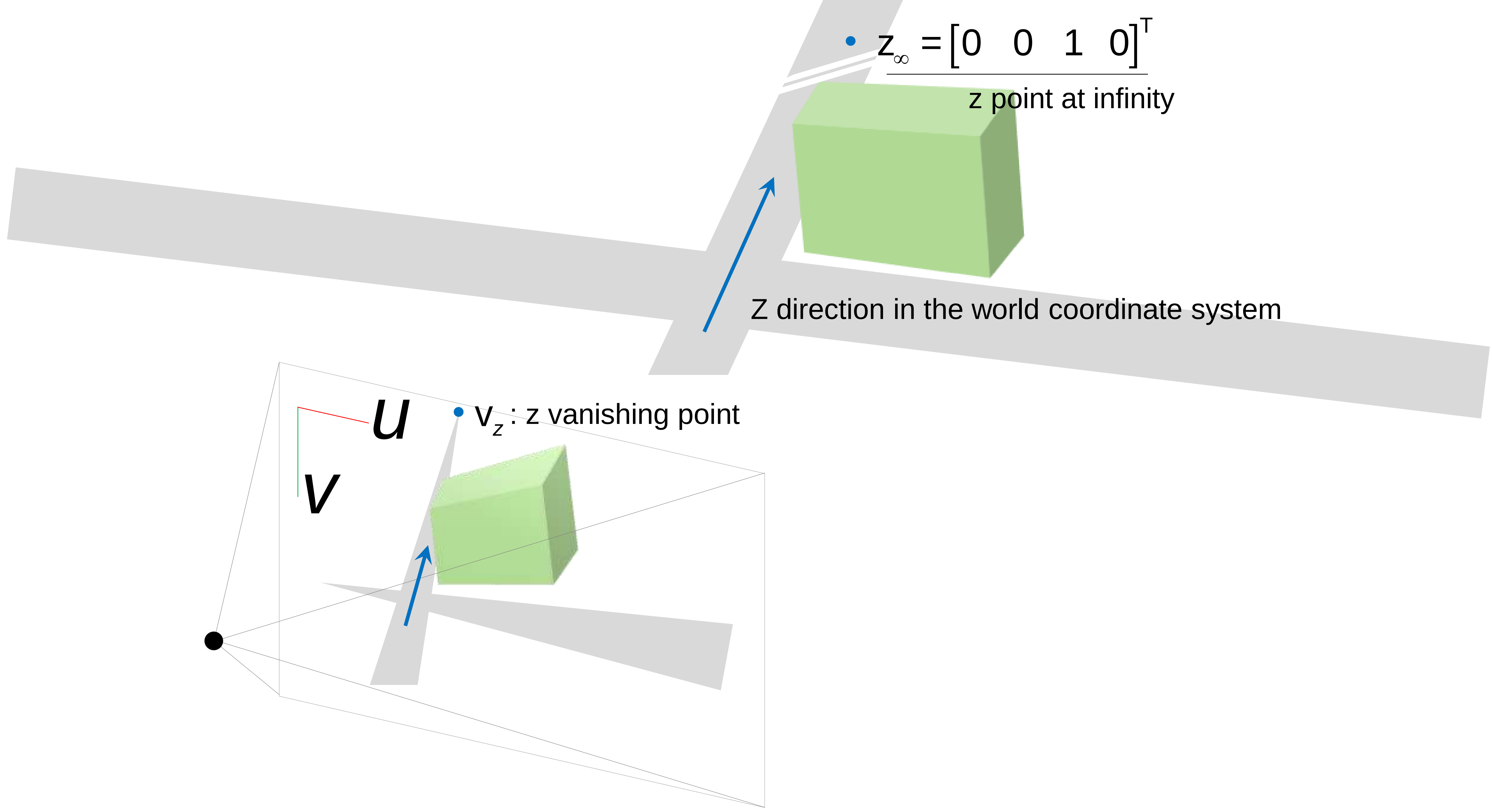
$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} -0.2374 & -0.9565 & 2.0138 & 1.2723 \\ 0.0578 & -1.5763 & 0.2404 & 1.2508 \\ 0.0004 & -0.0005 & 0.0007 & 0.0006 \end{bmatrix}$$

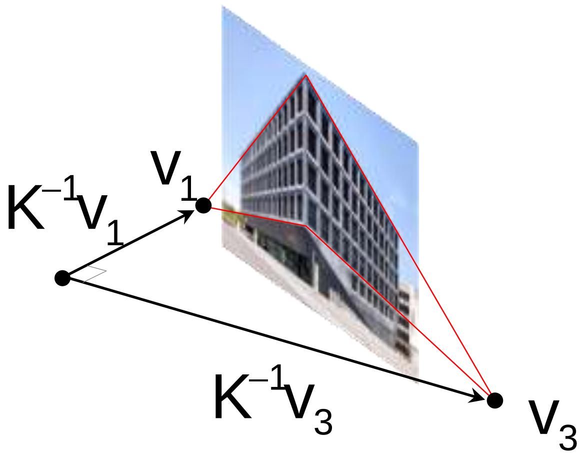
```
R = [-0.8496    0.0498    0.5731
   -0.3216   -0.8203   -0.4067
    0.4180   -0.5299    0.6835];

C = [0.0070
    0.7520
   -0.2738];

P = K * R * [eye(3) -C]
u_x = P(1:2,1)/P(3,1)
u_y = P(1:2,2)/P(3,2)
u_z = P(1:2,3)/P(3,3)
```

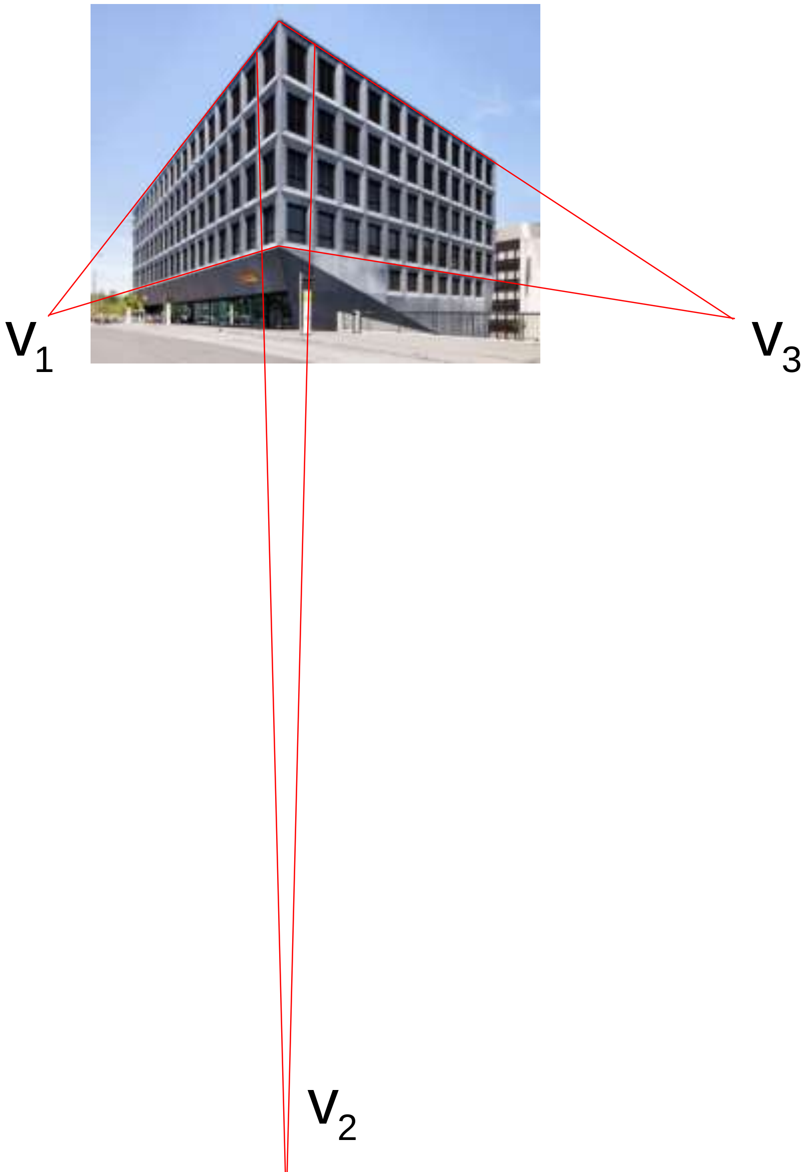

Z direction in the world coordinate system


Z direction in the world coordinate system
u
v



$z_\infty = [0 \;\; 0 \;\; 1 \;\; 0]^T$
z point at infinity
Z direction in the world coordinate system
v_z : z vanishing point
u
v

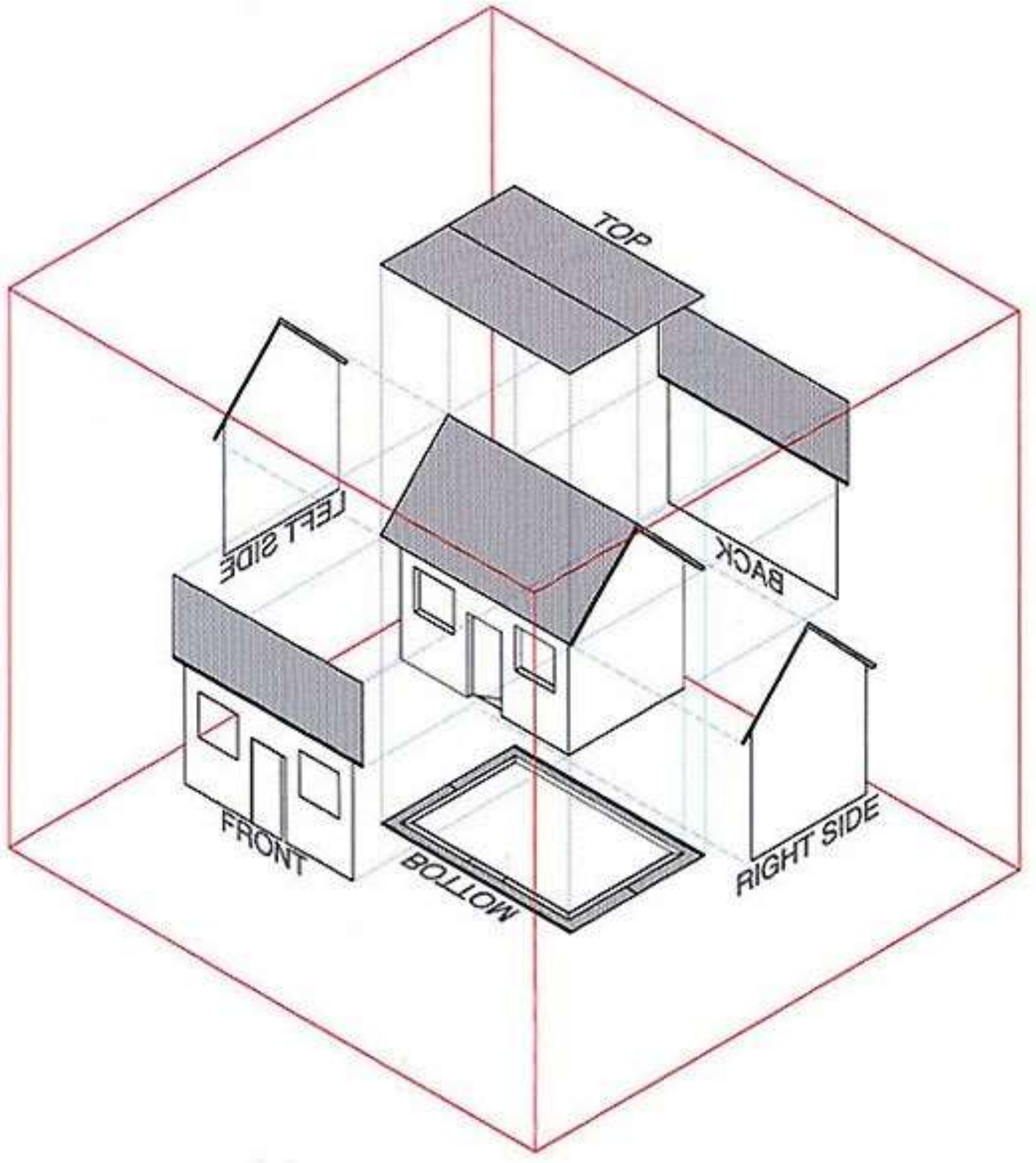

$K^{-1}v_1$
v_1
$K^{-1}v_3$
v_3


v_1
v_3
v_2

Weak perspectiveness

Strong perspectiveness

Orthographic Camera



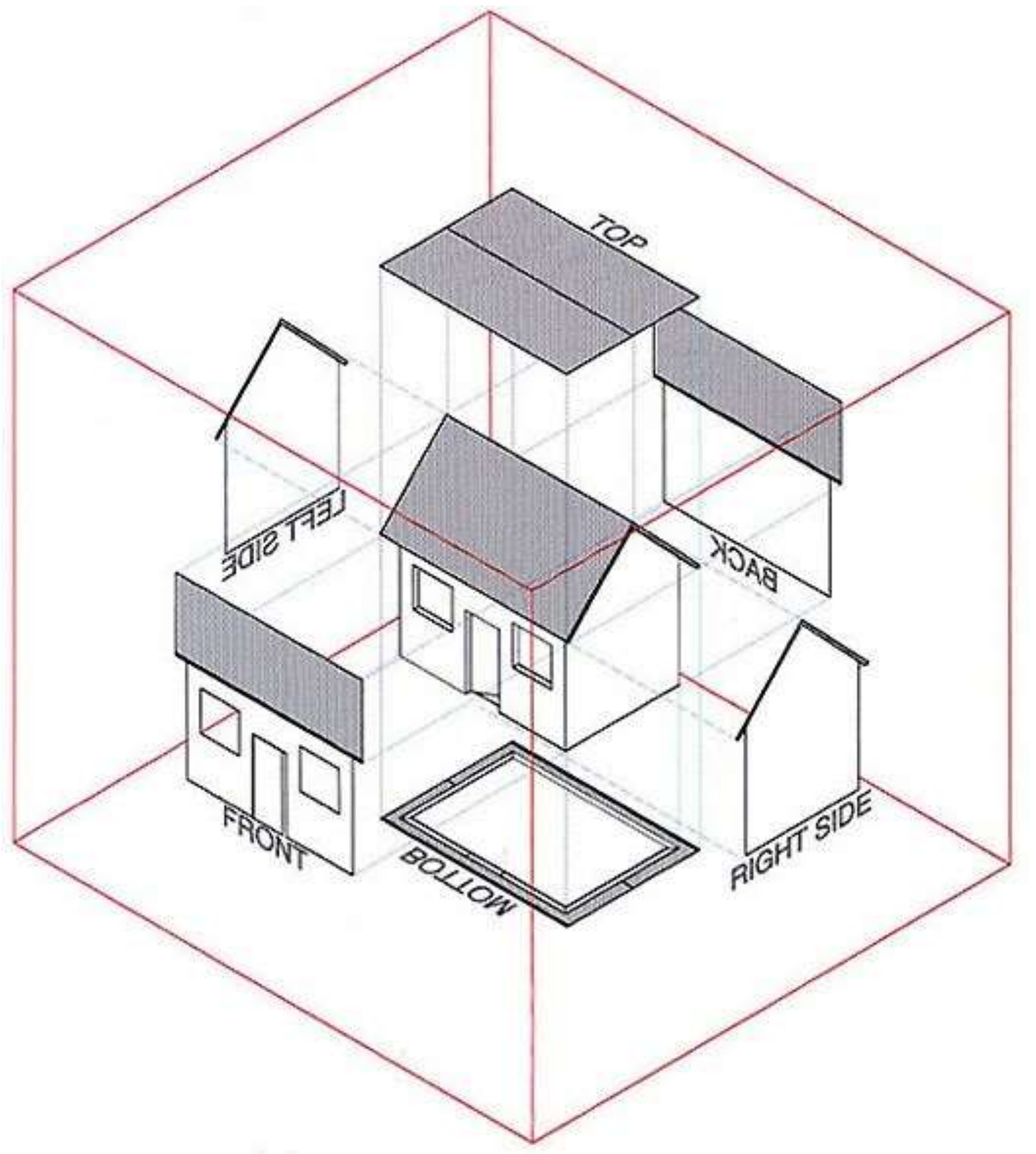
Affine camera:

$$P_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic camera:

$f = 1 \quad p_X = p_Y = 0$

Orthographic Camera



Affine camera:

$$P_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic camera:

$f = 1 \quad p_X = p_Y = 0$

$$P_O = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lens configuration (internal parameter)

$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = L\left(\mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \right)$$

Spatial relationship between sensor and pinhole (internal parameter)

Camera body configuration (extrinsic parameter)


Lens Radial Distortion