CIS 1921

Lecture 9: More Constraint Programming

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Logistics

- **HW4** due **Wednesday 4/2**
  - Get it out of the way so you can work on the project
  - Will be graded manually (lenient)
- **Project proposals** – feedback to be released soon
  - *Refer back to the feedback periodically while working*
- **Project checkpoint** due 4/10
  - Aim for ~75% completion

Recap: Constraint Programs

- Find an assignment of variables to values, subject to general constraints
- Discrete, finitely bounded domains (integers only)
- May or may not optimize an objective

The "if...then..." conundrum

- Suppose we want to implement logic along the lines of:

"If **x** condition holds, then **y** must hold"

i.e. we want a constraint to be tied to a variable / other constraint

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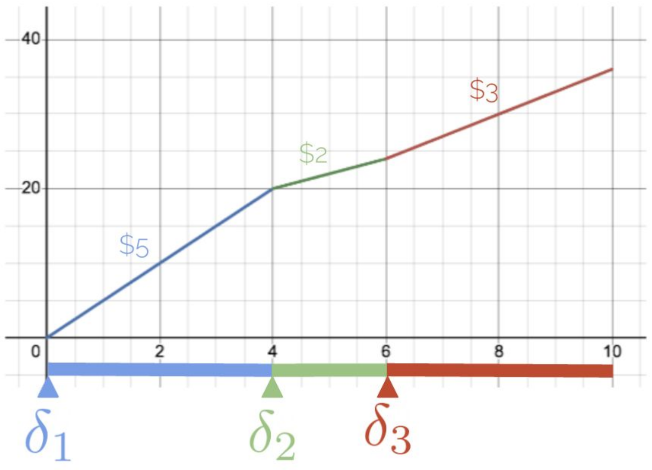

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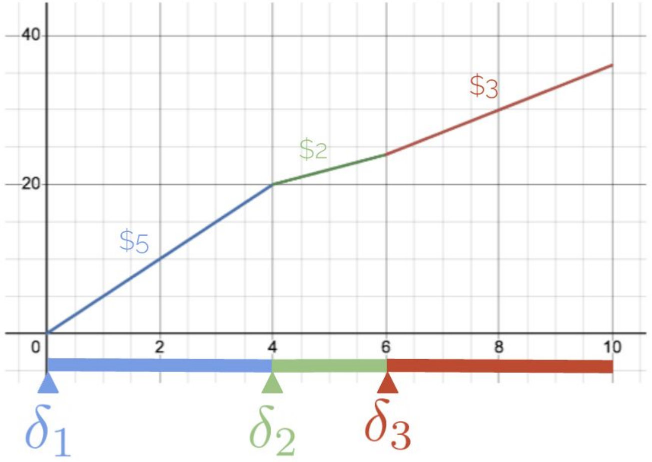

- If $i_1 = 0$, then $0 <= \delta_1 <= 400$
- If $i_1 = 1$, then $400 <= \delta_1 <= 400$

The "if…then…" conundrum

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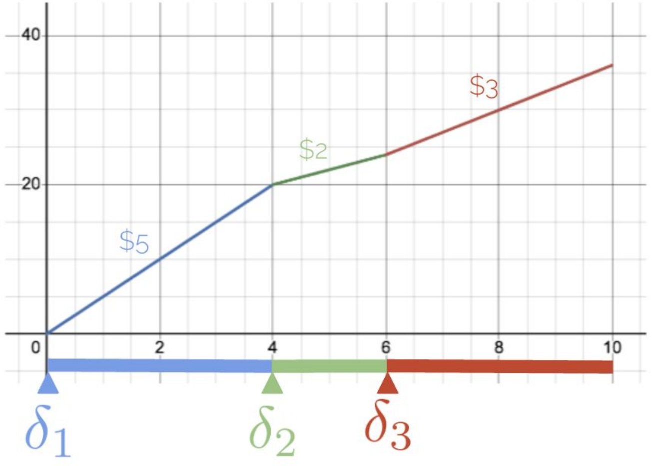


- If $i_1 = 0$, then $0 <= \delta_1 <= 400$
- If $i_1 = 1$, then $400 <= \delta_1 <= 400$

$$i_1 \cdot 400 \leq \delta_1 \leq 400$$

The "if...then..." conundrum

- Suppose we want to implement logic along the lines of:

  "If **x** condition holds, then **y** must hold"

- With Constraint Programming, our constraints aren't just linear inequalities, so we need a more general way of handling this!

Constraints for BoolVars

- Recall `model.NewBoolVar(name)`
  - Equivalent to `model.NewIntVar(0, 1, name)`
- `boolvar.Not()`
- `model.AddBoolOr(boolvars_list)`
- `model.AddBoolAnd(boolvars_list)`
- `model.AddImplication(b1, b2)`

Ex: Magic Sequence

- A **magic sequence** is a sequence $s_0, s_1, \dots, s_n$ where s_i = number of occurrences of i in the sequence
- Ex:

s_0	s_1	s_2	s_3	s_4
?	?	?	?	?

Ex: Magic Sequence

- A **magic sequence** is a sequence $s_0, s_1, \ldots, s_n$ where s_i = number of occurrences of i in the sequence
- Ex:

s_0	s_1	s_2	s_3	s_4
2	1	2	0	0

Ex: Magic Sequence

- A **magic sequence** is a sequence $s_0, s_1, \ldots, s_n$ where s_i = number of occurrences of i in the sequence

PROBLEM: Given n, does there exist a magic sequence s_0, s_1, …, s_n, and if so, what is it?

Ex: Magic Sequence

- A **magic sequence** is a sequence $s_0, s_1, \ldots, s_n$ where s_i = number of occurrences of i in the sequence

Step 1: Define the variables

Each s_i will be a variable.

Ex: Magic Sequence

- A **magic sequence** is a sequence $s_0, s_1, \dots, s_n$ where s_i = number of occurrences of i in the sequence

Step 2: Define the values for the variables

The minimum s_i can be is 0, the maximum is (n+1)

Ex: Magic Sequence

- A **magic sequence** is a sequence $s_0, s_1, \ldots, s_n$ where s_i = number of occurrences of i in the sequence

Step 3: Define the constraints for the problem.

S_i = # of occurrences of i amongst s_o, ..., s_n

"If the value i appears j times, then s_i = j"

Reification

- Allows us to express "if-then" relationships as constraints
  - Ex. "If x is equal to 5, then y must be greater than 7"
- **Reification:** the process of linking a logical condition to a boolean variable

- Introduce a new boolean (0/1) variable b which is true if and only if constraint c holds ($b \Leftrightarrow c$)
  - Essentially, name truth value of c with variable b

Reification

“If x is equal to 5, then y must be greater than 7”

- **Step 1:** Introduce a boolean variable which will indicate whether x = 5

```
is_x_five = model.NewBoolVar("is_x_five")
```

- **Step 2:** Tie the boolean indicator with the condition x = 5

$$x == 5 \iff \texttt{is\_x\_five} = 1$$

```
model.Add(x == 5).OnlyEnforceIf(is_x_five)
model.Add(x != 5).OnlyEnforceIf(is_x_five.Not())
```

- **Step 3**: Add further constraints with respect to the indicator:

```
model.add(y > 7).OnlyEnforceIf(is_x_five)
```

Reification in OR-Tools

- OR-Tools API uses **half-reification:** instead of $b \Leftrightarrow c$, just supports $b \Rightarrow c$
  - Can fully reify by combining $b \Rightarrow c$ and $\overline{b} \Rightarrow \overline{c}$

- `constraint.OnlyEnforceIf(bool_var)`
  - Means `bool_var ⇒ constraint`

Reification Warning

- `constraint.OnlyEnforceIf` only works for these constraints:
  - `Add`
  - `AddBoolOr`
  - `AddBoolAnd`
  - `AddLinearExpressionInDomain` (haven't seen this one yet)

- This is usually all you need

Magic Sequence in OR-Tools

- Initialize model and s_i variables

```
model = cp_model.CpModel()

# Create s_i variables
S = {}
for i in range(n+1):
    S[i] = model.NewIntVar(0, n+1, f's_{i}')
```

Magic Sequence in OR-Tools

- Reify constraints $s_i = j$ into new boolean variables

```
# Reified constraints: eq[i, j] <-> s_i == j
eq = {}
for i in range(n+1):
    for j in range(n+1):
        eq[i, j] = model.NewBoolVar(f's_{i} == {j}')
        model.Add(S[i] == j).OnlyEnforceIf(eq[i, j])
        model.Add(S[i] != j).OnlyEnforceIf(eq[i, j].Not())
```

Magic Sequence in OR-Tools

- Make s_i equal to number of occurrences of i

```python
# s_i = number of occurrences of i in sequence
for i in range(n+1):
    model.Add(
        S[i] == sum(eq[j, i] for j in range(n+1))
    )
```

Magic Sequence in OR-Tools

- Solve and print the output

```
solver = cp_model.CpSolver()
if solver.Solve(model) == cp_model.FEASIBLE:
    print([f's_{i}={solver.Value(S[i])}' for i in range(n+1)])
```

```
['s_0=2', 's_1=1', 's_2=2', 's_3=0', 's_4=0']
```

Non-contiguous Domains

- `cp_model.Domain.FromValues([0,2,4,6,8])`

0	1	2	3	4	5	6	7	8

- `cp_model.Domain.FromIntervals([0, 2],[6, 8])`

0	1	2	3	4	5	6	7	8

- `model.NewIntVarFromDomain(domain, name)`

Linear Expressions on Domains

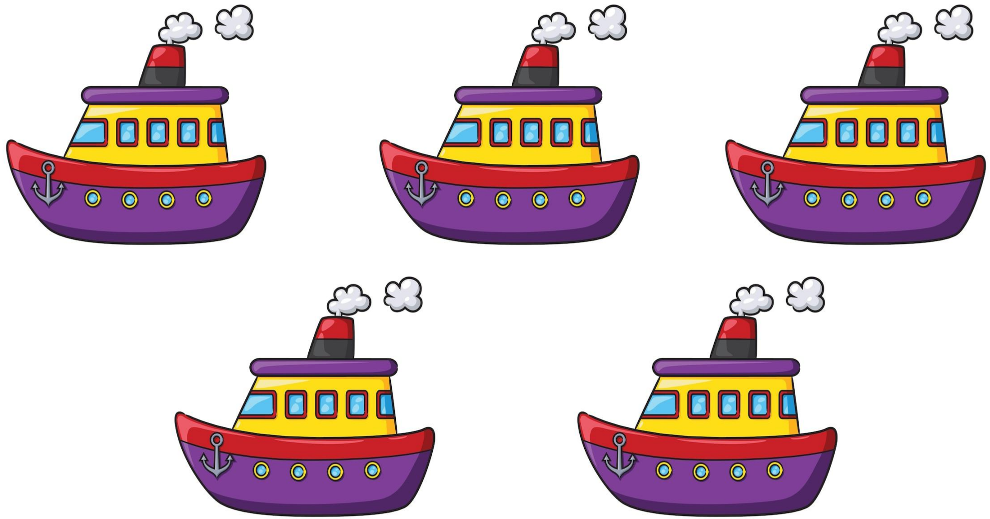
- Enforce that result of a linear expression must fall into a domain
- `cp_model.AddLinearExpressionInDomain(`

```
    x + y,
    cp_model.Domain.FromValues([0,2,4])
)
```

0,0	1,0	2,0	3,0	4,0
0,1	1,1	2,1	3,1	4,1
0,2	1,2	2,2	3,2	4,2
0,3	1,3	2,3	3,3	4,3
0,4	1,4	2,4	3,4	4,4

Ex: Shipping Allotments

- Shipping company has n ships with capacity 100 each
- Want to load all shipments of varying sizes onto ships
- **Goal:** maximize number of ships which have at least 20 capacity unused (in case of emergency)
    - See worked solution in additional code (ships.py)



0/100kg

0/100kg

20 kg	20 kg
20 kg	20 kg

15 kg	15 kg
15 kg	15 kg
15 kg	15 kg

30 kg	30 kg
30 kg	30 kg

45 kg	45 kg
45 kg	

75/100

15 kg	45 kg
15 kg	

75/100

30 kg	45 kg

75/100

15 kg	30 kg
	30 kg

100/100

20 kg	15 kg
20 kg	15 kg
30 kg	

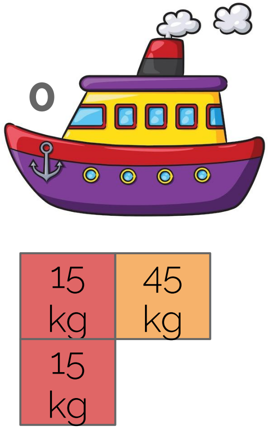
100/100

20 kg	15 kg
20 kg	45 kg

Solving the Problem

- Step 1: Define the variables

$n_{k,\,s}$ = number of boxes of size k on ship s



$n_{15,\,0} = 2$

$n_{20,\,0} = 0$

$n_{30,\,0} = 0$

$n_{45,\,0} = 1$

Solving the Problem

- Step 2: Define the values for the variables

$n_{k,\,s}$ = number of shipments of size k on ship s

$n_{k,\,s}$ >= 0

$n_{k,\,s}$ <= # of size k boxes we have

Solving the Problem

- Step 3: Define the constraints

  $n_{k,s}$ = number of shipments of size k on ship s

  - Each box is on exactly one ship

$$\text{For each “box” k ,} \quad \sum_i n_{k,\text{ship } i} == \text{count}_k$$

  - We do not exceed the capacity of a ship

$$\text{For each ship } s, \quad \sum_k k \cdot n_{k,s} \leq \texttt{capacity}$$

Solving the Problem

- Step 4: Include Objective
    - Maximize the number of ships with 20 free capacity

count_cap

For each ship, **if** it has 20 free capacity, **then** it contributes to **count_cap**

Reification!
(go to code)

Tuning the CP-SAT Solver

- We can play around with CP-SAT internals to possibly speed up the search
- There are tons of parameters that can be adjusted
  - Some are documented better than others…
  - https://github.com/google/or-tools/blob/stable/ortools/sat/sat_parameters.proto
- **Warning:** these things are generally far less important than having a good encoding

Parallelization

- We can run solver computation in parallel across multiple threads

```
solver = cp_model.CpSolver()
solver.parameters.num_search_workers = 4
```

  - By default, CP-SAT will try to use all available cores

Hinting

- We can give the model a **hint** to try setting a variable to a specified value

```
# try setting x = 5 first
model.AddHint(x, 5)
```

Quick & Dirty Optimization

- Finding an optimal solution can take far longer than finding a feasible solution
- Often in practice, we don't *really* care about having the true optimal value with total certainty
    - Just want it to be "close enough"

Quick & Dirty Optimization

Solution:

- Optimize objective and run solver for a reasonable amount of time (depends on your patience)
- Interrupt early with Ctrl+C or `max_time_in_seconds` param
  - If interrupted, solver returns FEASIBLE instead of OPTIMAL
- Print the intermediate objective value and solution and decide if it's "good enough"
  - For tough problems, no guarantee that you are close to optimal!
  - `best_bound` in response stats gives best LB (when minimizing) or UB (when maximizing) proved so far for optimal value

Quick & Dirty Optimization

- Helpful: set `log_search_progress` param to True
  - Prints every time a new best solution is found
- Sometimes helpful: custom solution callback
  - Called each time any new feasible solution is found

```
class BestSolutionFinder(cp_model.CpSolverSolutionCallback):

    def __init__(self, minimizing=True):
        cp_model.CpSolverSolutionCallback.__init__(self)
        self.minimizing = minimizing
        self.best_value = (1 if minimizing else -1) * float('inf')

    def on_solution_callback(self):
        obj = self.ObjectiveValue()
        if (self.minimizing and obj < self.best_value) \
        or (not self.minimizing and obj > self.best_value):
            self.best_value = self.ObjectiveValue()
            print(f'New best value: {self.best_value}')
```

```
solver = cp_model.CpSolver()
solver.parameters.num_search_workers = 6
solver.parameters.log_search_progress = True
# Our solution callback is redundant to logging
best = BestSolutionFinder()
solver.SolveWithSolutionCallback(model, best)
```

Approximating Feasibility

- What if non-optimization problem is too hard to solve?
- Can't interrupt early for a "good enough" solution; intermediate solution is feasible or it is not
- What if we were OK with a "not quite feasible" solution?
    - What could "not quite feasible" mean?

Soft Constraints

- Constraints like `Add(...)` are **hard constraints**
  - Must be satisfied
- **Soft constraints**: can be violated, but incurs a penalty
- Transform feasibility problem into optimization problem by minimizing penalty
  - Allows interrupting early if you're OK with some violated constraints
  - Can sometimes be faster than solving with hard constraints!

Ex: Soft Graph Coloring

- Hard constraint:

  for every edge (u, v), $color(u) \neq color(v)$

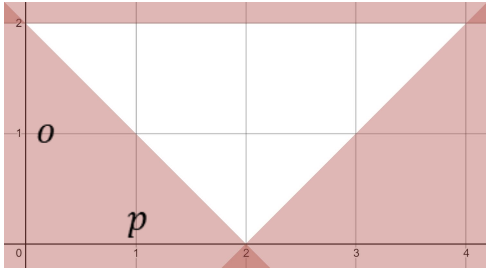
- Soft constraint

  $penalty$ = num. edges (u, v) with $color(u) = color(v)$

- Can count number of violated constraints using reification

Optimizing Pairs of Objectives

- What if we want to add soft constraint with penalty p but problem already optimizes (say, minimizes) objective o?
- **Key idea:** why not minimize both?
- Attempt 1: minimize $\boldsymbol{o} + \boldsymbol{p}$
  - **Problem:** o and p may be interrelated
  - E.g., minimum possible value of o may be lower when $p = 1$ than when $p = 0$



Optimizing Pairs of Objectives

- **Observation:** avoid interdependence by minimizing p first and using o to break ties
  - Aka, minimize (p, o) over the **lexicographic ordering**
- How to make sure that p is minimized before o?
- Attempt 2:
  - minimize $\boldsymbol{Mp + o}$, where $M = o_{max} - o_{min} + 1$
- Can generalize to maximization

General CP-SAT Modeling Tips

- Don't be afraid to add new variables/constraints, but be aware of roughly how many you have ($O(n)$? $O(n^3)$?)
- Try to restrict range of values for each variable
- Use boolean variables/constraints when possible
- Experiment with hard vs. soft constraints
- If possible, split into subproblems, then combine solutions
- Make it easy to toggle constraints on/off for debugging

MIP vs CP-SAT

MIP	CP-SAT
• Supports infinite bounds • Supports fractional variables and coefficients • Better handles LP-style problems (with integers mixed in) • Reification of constraints is possible, but requires algebraic modeling trick	• Better handles combinatorial problems, Booleans • More sophisticated interface • Lots of specialized modeling objects • Modeling may be easier • Models may be more extensible • Reification is easier, more performant

- Neither is clearly more performant in general
- Neither is an evolution of the other