

CIS 1600 Recitation 9

Variance, Graph Colorings

October 31 and November 1, 2024

Variance

- ▶ How much a random variable deviates from its mean.
- ▶ The variance of a random variable X is defined as

$$\text{Var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

- ▶ The standard deviation of a random variable X is

$$\sigma[X] = \sqrt{\text{Var}[X]}$$

Properties of Variance

- ▶ For random variable X in the sample space Ω and $c \in \mathbb{R}$,
$$\text{Var}[cX] = c^2 \text{Var}[X]$$
- ▶ If X and Y are independent random variables, then
$$\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y] \text{ and } E[XY] = E[X]E[Y]$$

Markov's Inequality

Let X be a non-negative random variable. For all $a > 0$:

► **Markov's Inequality**

$$\Pr[X \geq a] \leq \frac{E[X]}{a}$$

Graph coloring

- ▶ A graph is *k-colorable* if each vertex can be colored using one of the k colors so that adjacent vertices are colored using different colors.
- ▶ The *chromatic number* of a graph G , $\chi(G)$, is the smallest value of k for which G is k -colorable
- ▶ A *bipartite* graph is a graph that is 2-colorable.
- ▶ A graph with maximum degree at most k is $(k + 1)$ -colorable.