

CIS 1600 Recitation 8

Expectation, Rooted Trees

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Expectation

- ▶ The weighted average (proportional to the probabilities) of the possible values of X .
- ▶ $\mathbb{E}[X]$ is the value we would expect to obtain if we repeated a random experiment many times and took the average of the outcomes of X .

$$\mathbb{E}[X] = \sum_i i \cdot Pr[X = i]$$

Linearity of Expectation (LOE)

- ▶ The expectation of the sum of random variables equals the sum of their expectations.
- ▶ For random variables X_1, X_2, \dots, X_n on the same probability space Ω and $c_1, c_2, \dots, c_n \in \mathbb{R}$

$$E\left[\sum_{i=1}^n c_i X_i\right] = \sum_{i=1}^n c_i E[X_i]$$

- ▶ Random variables do not have to be independent.

Rooted Trees

- ▶ A tree in which one vertex is distinguished from the others and is called the **root**
- ▶ The **level** of a vertex v is the number of edges along the unique path between v and the root
- ▶ The **height** of a rooted tree is the maximum level of any vertex in the tree

Rooted Trees Continued

- ▶ The **children** of v are neighbors of v that are one level away from the root than v . If u is a child of v , then v is the **parent** of u . If two vertices share the same parent, they are **siblings**.
- ▶ Given vertices v and w , if v lies on the unique path between w and the root, then v is an **ancestor** of w and w is a **descendant** of v .

Eulerian & Hamiltonian Graphs

- ▶ An Eulerian circuit of a graph G is a closed walk in which each edge of G appears exactly once.
- ▶ A connected graph is Eulerian if it contains an Eulerian circuit.
- ▶ A connected graph G is Eulerian iff every vertex has even degree.
- ▶ A Hamiltonian cycle in a graph G is a cycle in which each vertex of G appears exactly once.
- ▶ A graph is Hamiltonian if it contains a Hamiltonian cycle.