# CIS 1600 Recitation 5

## Intro to Graphs, Intro to Probability

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# Graph Terminology

- Undirected graph: $G = (V, E)$ where
- V: finite, non-empty set of **vertices**
- E: finite (possibly empty) set of **edges**
- An edge $\{u, v\}$ connects vertices $u$ and $v$.

# Graph Terminology - Continued

- Two vertices $u, v$ are **adjacent** if $\{u, v\} \in E$.
- Nodes adjacent to a vertex $u$ are called **neighbors** of $u$.
- **Degree** of a vertex, $deg(u)$ is the number of neighbors of $u$.
- $\delta(G) = min_{v \in V} deg(v)$ is the minimum degree of G
- $\Delta(G) = max_{v \in V} deg(v)$ is the maximum degree of G.

# Graph Lemmas

- The Handshaking Lemma: the sum of the degrees of all vertices in a graph is twice the number of edges

$$\sum_{v \in V} deg(v) = 2|E|$$

- In any graph, there are an even number of vertices of odd degree

# Intro to Probability

- The sample space $\Omega$ is the set of all possible outcomes.
- The probability space is a sample space together with a probability distribution assigned to each outcome $\omega \in \Omega$ s.t.

$$0 \leq Pr[\omega] \leq 1$$

$$\sum_{\omega \in \Omega} Pr[\omega] = 1$$

- A subset of the sample space is called an event.
- For any event $A \subseteq \Omega$, the probability of $A$ is defined as:

$$Pr[A] = \sum_{\omega \in A} Pr[\omega]$$

# Uniform Probability Space

- A probability space $(\Omega, \Pr)$ is uniform if all outcomes have the same probability
- $\Pr[\omega] = \frac{1}{|\Omega|}$, for all $\omega \in \Omega$
- $\Pr[E]=\frac{|E|}{|\Omega|}$, for some event $E \subseteq \Omega$

# Steps to Solve Probability Problems

1. Define a sample space $\Omega$ of the experiment.
2. Define the probability distribution.
3. Find the event of interest $A$ (subset of outcomes $A \subseteq \Omega$ that are of interest).
4. Compute $Pr[A]$ by adding up probabilities of the outcomes in $A$.

# The Inclusion-Exclusion Formula

- If $A, B, C$ are any events,

$$\begin{aligned} Pr[A \cup B] &= Pr[A] + Pr[B] - Pr[A \cap B] \\ Pr[A \cup B \cup C] &= Pr[A] + Pr[B] + Pr[C] \\ &\quad - Pr[A \cap B] - Pr[A \cap C] - Pr[B \cap C] \\ &\quad + Pr[A \cap B \cap C] \end{aligned}$$

- Union-bound

$$Pr[\cup_{i=1}^{n} A_i] \leq \sum_{i=1}^{n} Pr[A_i]$$

If the events are pairwise disjoint, the inequality becomes equality.