CIS 1600 Recitation 4

Induction, Binomial Theorem, Pigeonhole Principle

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Proof Technique: Mathematical Induction

Let $P(n)$ be a predicate whose truth depends on n. We want to show that it is true $\forall n \geq n_0$

- Base Case: Check that $P(n_0)$ is true.
- Induction Hypothesis: Assume $P(k)$ is true for some *arbitrary but particular* integer $k \geq n_0$.
- Induction Step: Show that $P(k+1)$ is true. Conclude $P(n)$ for all $n \geq n_0$, where $n \in \mathbb{Z}$

Strong Induction

- Let n_0 be a natural number and let $P(n)$ be a predicate for all natural numbers $n \geq n_0$.
- Base Case: $P(n_0)$ holds
- Induction Hypothesis: $P(j)$ is true for $n_0 \leq j \leq k$
- Induction Step: We want to show that $P(k+1)$ is true.
  That is, for any $k \geq n_0$,
  $P(n_0) \land P(n_0+1) \land ... \land P(k) \implies P(k+1)$ is true.
- Strong and ordinary induction are mathematically equivalent.

The Binomial Theorem

- The binomial theorem gives an expression for $(a+b)^n$
- For any real numbers a and b and non-negative integer n

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

The Pigeonhole Principle

- $k + 1$ or more objects are distributed among k bins
- There is at least one bin that has two or more objects.

Generalized Pigeonhole Principle:

- n objects are placed into k bins
- There exists at least 1 bin containing at least $\lceil \frac{n}{k} \rceil$ objects.