CIS 1600

Recitation Guide - Week 4

Topics Covered: Induction, Pigeonhole Principle, Binomial Theorem

Problem 1:

Suppose we have the following sequence:

$$a_1 = 1 \qquad a_2 = 3 \qquad a_i = a_{i-2} + 2a_{i-1}, \quad i \in \mathbb{Z}, i \geq 3$$

Use induction to prove that for all integers $n \geq 1$, a_n is odd.

Problem 2: Let S be a set of 16 distinct positive integers such that $\forall x \in S$, $x < 60$. Show that there exists distinct integers $a, b, c, d \in S$ such that $a + b = c + d$.

Problem 3: Our favorite head TAs Andrew and Eric are playing a game in which there are two non-empty bags with an equal number of marbles in them. In this game, the two players take turns removing marbles from one of the bags. In each turn, the player can remove any positive number of marbles as long as they are all from the same bag. The winner of the game is the player that removes the last marble. In Andrew and Eric's current configuration, both bags initially start with the same number of marbles. Prove that one of them can guarantee to always win.

Problem 4:

All the sheep in Winston's flock have the same color! Winston claims that he can use induction to prove all sheep in the world have the same color. Find the fault in his reasoning.

Base Case: size = 1. One sheep, one color. ✓

Induction Hypothesis: Assume that in a flock of size k, where $k \in \mathbb{Z}^+$, all sheep have the same color.

Induction Step: We want to prove the claim is true for a flock of size $k + 1$. Take one sheep, let's call her Sara, out. What remains is a flock of size k, so by IH, they all share the same color. Now put Sara back in and take out another sheep, let's call him Thomas Zeuthen, out. By IH, what remains is a flock of size k, so by IH, they all share the same color. Sara and Thomas Zeuthen must share the same color as they both are the same color as the other $k - 1$ sheep. Thus we arrive at the conclusion that all $k + 1$ sheep share the same color.