CIS 1600 Recitation 3

Permutations, Combinations, Inclusion-Exclusion, Mathematical Induction

September 12-13, 2024

The Inclusion-Exclusion Formula

- If A, B, C are any finite sets,

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

- If A, B, C are mutually disjoint ($A \cap B = A \cap C = B \cap C = \emptyset$)

$$|A \cup B| = |A| + |B|$$
$$|A \cup B \cup C| = |A| + |B| + |C|$$

This is called the *addition rule*.

Multisets

An unordered collection where repetitions are allowed.

- Permutation of Multiset: n_k are of type a_k and indistinguishable from one another.

$$\{n_1 \cdot a_1, n_2 \cdot a_2, \ldots, n_k \cdot a_k\}$$

- Number of Permutations:

$$\frac{n!}{n_1! n_2! \cdots n_k!}$$

Where $n = n_1 + n_2 + \cdots + n_k$

Sticks and Crosses

- The number of ways to distribute r indistinguishable objects into n distinguishable containers.
- r crosses
- $n-1$ sticks (you only need $n-1$ sticks to divide something into n groups)

$$\binom{n+r-1}{r} = \binom{n+r-1}{n-1} = \frac{(n+r-1)!}{(n-1)!r!}$$

Combinatorial Proofs

Asked to prove an identity.

- Pose a counting question.
- Answer the question in one way, giving us the LHS.
- Answer the question in another way, giving us the RHS.
- Since the LHS and RHS answer the same question, they must be equal.
- No algebraic manipulation of the identity should be done throughout the proof.