CIS 1600 Recitation 15

Functions and Planar Graphs

December 5-6, 2024

Functions

$f : A \rightarrow B$

- ▶ Set A: domain

- ▶ Set B: codomain

- ▶ A relation where each element of the domain $x \in A$ is related to exactly one element of the codomain $f(x) \in B$

Range of function $f : A \rightarrow B$

- ▶ $Ran(f) = \{y \mid y \in B \land \exists x \in A \, s.t \, y = f(x)\}$

Injections and Surjections

- $f : A \to B$ is **injective** if it maps distinct elements to distinct elements i.e for every $x_1 \neq x_2$ in domain, we have $f(x_1) \neq f(x_2)$

- $f : A \to B$ is **surjective** if $Ran(f) = B$ or $\forall y \in B$, $\exists x \in A \; s.t. \; y = f(x)$

Bijections

- $f : A \to B$ is bijective if it is both injective and surjective

- Bijection rule: if we can define a bijective function with domain A and codomain B, $|A| = |B|$

Planar Graphs

A **planar graph** G is a graph with at least one crossing free embedding (edges do not intersect themselves or other edges, except at endpoints)
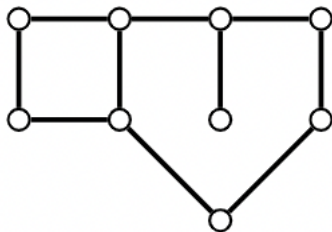
Euler's Formula

- **Face**: a region of the plane cut off from other regions by edges

- Let G be a connected planar graph with n vertices and m edges. For any crossing-free embedding of G,

$$n - m + f = 2$$

  where f is the number of faces

- **Corollary:** Let $G = (V, E)$ be a planar graph with at least two edges, then $m \leq 3n - 6$.

Other Properties of Planar Graphs

- ▶ The minimum degree in a planar graph is at most 5.

- ▶ Every planar graph is 4-colorable.