CIS 1600 Recitation 11

Hall's Theorem, Relations

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Hall's Theorem

- Let $G = (X, Y, E)$ be a bipartite graph. For any set S of vertices, let $N_G(S)$ be the set of vertices adjacent to vertices in S.
- G contains a matching that saturates every vertex in X iff $|N_G(S)| \geq |S|, \forall S \subseteq X$. (Hall's condition)

Relations

- A binary relation is a set of ordered pairs.
- For example, $R = \{(1,2),(2,3),(5,4)\}$
- $(1,2) \in R$: 1 is related to 2 by relation R, we denote this by $1R2$.
- A binary relation R from set A to B is a subset of the Cartesian product $A \times B$.

Properties of Relation

- *Reflexive:* for all $x \in A, (x, x) \in R$.
- *Irreflexive:* for all $x \in A, (x, x) \notin R$.
- *Symmetric:* for all $x, y \in A, (x, y) \in R \implies (y, x) \in R$.
- *Antisymmetric:* for all $x, y \in A, (x, y) \in R$ and $(y, x) \in R \implies x = y$.
- *Transitive:* for all $x, y, z \in A, (x, y) \in R$ and $(y, z) \in R \implies (x, z) \in R$.
- *Symmetric* and *antisymmetric* are not opposites.