CIS 1600
Recitation Guide - Week 11

Topics Covered: Relations, Hall's Theorem

Problem 1:
Consider a normal chessboard (an 8×8 grid). In each row and in each column there are exactly n pieces, where $0 < n \leq 8$. Prove that we can pick 8 pieces such that no two of them are in the same row or column.

Problem 2:
Define an equivalence relation R on the set $\{1, 2, 3, \ldots, 100\}$ with the restriction that there are exactly 2 equivalence classes. Find an R such that it maximizes the size of the relation, and then show that the size is maximized.

Problem 3:

Consider a set A with $n \geq 1$ elements. We color independently each of the elements of A red with probability $\frac{1}{3}$ and blue with probability $\frac{2}{3}$. Let R be the "is the same color as" relation on A, ie. if a is the same color as b, then $(a, b) \in R$.

a) Is R an equivalence relation? If so, what are its equivalence classes?

b) Calculate the expected value of $|R|$.