CIS 1600 Recitation 10

Binomial and Geometric Distribution, Relations, Memoryless Property, Chebyshev's Inequality

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Binomial Distribution

- A sequence of n Bernoulli trials that are independent and each has a probability p of success. How many successful outcomes?
- Example: A sequence of n coin flips in which the probability of obtaining heads is p. How many flips result in head?
- A binomial r.v. X with parameters n and p has the following distribution for $j = 0, 1, 2, ..., n$:

$$Pr[X = j] = \binom{n}{j} p^j (1-p)^{n-j}$$

- $E[X] = np$ and $Var[X] = np(1-p)$

Geometric Distribution

- A sequence of Bernoulli trials that are independent with each having a probability p of success, that stops **after the first success**
- Example: A sequence of coin flips in which the probability of obtaining heads is p. How many flips until we reach our first head?
- $\Omega = \{H, TH, TTH, TTTH, ...\}$
- For any $\omega \in \Omega$ of length i, $Pr[\omega] = (1-p)^{i-1}p$.

Geometric Distribution

- A geometric r.v. X with parameter p has the following distribution for $i = 1, 2, ...$

$$Pr[X = i] = (1 - p)^{i-1} p$$

- $E[X] = \frac{1}{p}$ and $Var[X] = \frac{1-p}{p^2}$
- **Memoryless Property.** For geometric r.v. X with parameter p and for $n > 0$ and $k \geq 0$,

$$Pr[X = n + k \mid X > k] = Pr[X = n]$$

Hall's Theorem

- Let $G = (X, Y, E)$ be a bipartite graph. For any set S of vertices, let $N_G(S)$ be the set of vertices adjacent to vertices in S.
- G contains a matching that saturates every vertex in X iff $|N_G(S)| \geq |S|, \forall S \subseteq X$. (Hall's condition)

Chebyshev's Inequality

Let X be a random variable. For all $a > 0$:

$$Pr\Big[|X - E[X]| \geq a\Big] \leq \frac{Var[X]}{a^2}$$