CIS 1600

Recitation Guide - Week 1

Topics Covered: Multiplication rule, Parity, Sets

Problem 1: You own three different rings. You wear all three rings, but no two of the rings are on the same finger, nor are any of them on your thumbs. In how many ways can you wear your rings? (Assume any ring will fit on any finger and you may wear these rings on the fingers of both hands.)

Solution:

We can apply the multiplication rule.

Step 1: Choose the first finger. (8 ways)
Step 2: Choose the second finger. (7 ways)
Step 3: Choose the third finger. (6 ways)

Multiplying all of these, we have $8 \times 7 \times 6 = \boxed{336}$.

Problem 2: Show that for any two integers m and n, $m^2 + n^2$ has the same parity as $m + n$.

Solution:

Consider the following two cases which cover all possibilities.

Case 1: $m + n$ is odd

We can write $m + n = 2k + 1$, for some $k \in \mathbb{Z}$. Then, we have,

$$\begin{aligned} m^2 + n^2 &= (m + n)^2 - 2mn \\ &= (2k + 1)^2 - 2mn \\ &= 4k^2 + 4k + 1 - 2mn \\ &= 2(2k^2 + 2k - mn) + 1 \end{aligned}$$

which is odd, as $2k^2 + 2k - mn \in \mathbb{Z}$.

Case 2: $m + n$ is even

We can write $m + n = 2k$, for some $k \in \mathbb{Z}$. Similarly, we have,

$$\begin{aligned} m^2 + n^2 &= (m + n)^2 - 2mn \\ &= (2k)^2 - 2mn \\ &= 4k^2 - 2mn \\ &= 2(2k^2 - mn) \end{aligned}$$

which is even, as $2k^2 - mn \in \mathbb{Z}$.

We have shown that $m + n$ has the same parity as $m^2 + n^2$ and we are done.