CIS 1600 Recitation 1

Intro to Logic, Proofs

August 29-30, 2024

Proposition and Compound Propositions

A proposition is a statement that is either true or false.

- **Negation:** $\bar{p}$ (not p)
- **Conjunction:** $p \wedge q$ (p and q)
- **Disjunction:** $p \vee q$ (p or q)
- **Exclusive Or:** $p \oplus q$ (p exclusive-or q)
- **Implication:** $p \implies q$ (p implies q)
- **Biconditional:** $p \iff q$ (p if, and only if, q)
- $p \to q$: p is a *sufficient* condition for q.
- $\neg p \to \neg q \equiv q \to p$: p is a *necessary* condition for q.

Truth Table and Logical Equivalence

Truth Table

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T							
T	F							
F	T							
F	F							

Logical Equivalence

- Two compound propositions are logically equivalent if they always have the same truth value.
- Can be proved by the truth tables or a sequence of previously derived logically equivalent statements.

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Predicates and Quantifiers

- A predicate $P(x)$ contains a variable and becomes a proposition when the variable is assigned a value (e.g., $x < 5$)
- **Universal Quantifier:** $\forall$ ("for all") alongside $P(x)$ means $P(x)$ is true for all elements in the domain of x. (e.g., $\forall x \in \mathbb{Z}, x^3 + 1$ is composite.)
- **Existential Quantifier:** $\exists$ ("there exists") alongside $P(x)$ means there exists an element in the domain of x for which $P(x)$ is true. (e.g., $\exists x \in \mathbb{N}, x^2 \leq x$)