No office hours today.

Ex: # ways to order 26 letters of the alphabet so that no two of the vowels a, e, i, o, u occur consecutively?

Soln: The procedure of constructing such an arrangement is as follows.

S_1: Arrange the consonants.

S_2: Place the vowels in the 22 distinct spots between & around the consonants.

S_{2a}: find a posn for 'a' | 22

S_{26} : " " " " 'e' | 21

'i' ⋮

'o'

'u'

\# ways to do step 1 : $21!$

\# ways to do Step 2 : $P(22,5)$

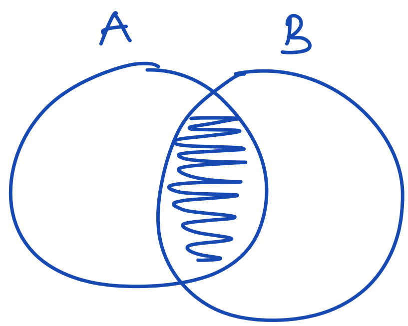
By the MR, # arrangements $= 21! \cdot P(22,5)$.

_ m _ b _ h _ d _ q _ r _ p _ s _

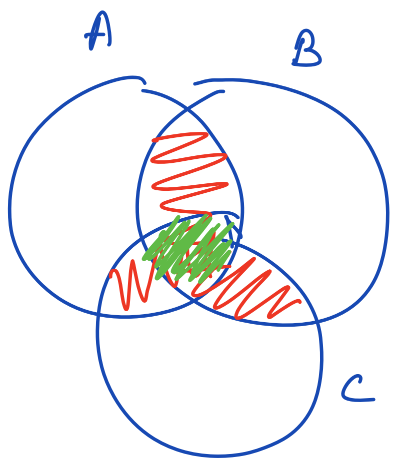
1 2 3 .

a e i o u .

Inclusion - Exclusion.



$$|A \cup B| = |A| + |B| - |A \cap B|.$$



$$|A \cup B \cup C| = |A| + |B| + |C|$$
$$- |A \cap B| - |A \cap C| - |B \cap C|$$
$$\cancel{- |A \cap B \cap C|} + |A \cap B \cap C|$$
$$\cancel{+ 3|A \cap B \cap C|}.$$

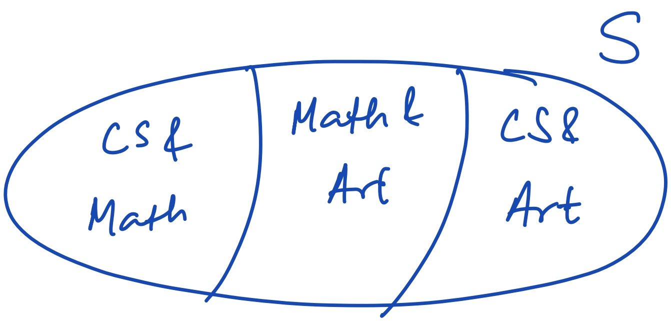
$$|A \cup B \cup C \cup D|$$

If A & B are disjoint then

$$|A \cup B| = |A| + |B|.$$ ← Addition rule.

Ex : # ways to select two distinct books from different subjects among five distinct CS books, three distinct math books & two distinct art books.

Soln :



Let S be the set of all pairs of books where the books are chosen from different subjects.

Let S_1 be the set of all pairs of books where one book is from CS & the other from Math.

S_2 : ____________ CS & one from Art

S_3 : ____________ M , Art.

Clearly, S_1, S_2, S_3 partition the set S.

$$\therefore |S| = |S_1| + |S_2| + |S_3|$$

$= 15 + 10 + 6$

$= \underline{\underline{31}}$

Ex: # integers b/w 1 & 1000 that are multiples of 3 or multiples of 5?

<u>Soln</u>: Let S_3 be the int b/w 1 & 1000 that are multiples of 3.

S_5 : ______________________ multiples of 5.

$|S_3| = 333$

$|S_5| = 200$

Our ans $= |S_3| + |S_5| = 533$ ✓

S_3 & S_5 are not disjoint.

$\therefore \; |S_3 \cup S_5| = |S_3| + |S_5| - |S_3 \cap S_5|$

$= 533 - |S_{15}|$

$= 533 - 66.$

15 990

15·1 15·66

Ex: # 4-letter strings that contain at least one x?

Soln: The proc. of constructing a 4-letter string containing one x is as follows:

S₁: Choose a posn for x | 4

S₂: Choose letters for the other three positions. | 26^3

By the MR, our answer $= 4 \times 26^3$. ✓

$\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$
1 2 3 4

$\{x, a\}$

$\boxed{\underline{x}\ \underline{x}}$

$x\ x$

$\underline{x}\ \underline{a}$

$x\ a$

$\underline{a}\ \underline{x}$

$a\ x$

$\boxed{\underline{x}\ \underline{x}}$

exactly x 1 | 2 . . . .

S : Set of all 4-letter strings

S_x: ______________________ containing x.

$S_{\bar{x}}$: ______________________ not containing x.

Clearly, S_x & $S_{\bar{x}}$ partition S.

$$|S| = |S_x| + |S_{\bar{x}}|.$$

$$|S_x| = |S| - |S_{\bar{x}}|$$

$$= \boxed{26^4 - 25^4}$$

Ex: # even digit integers with distinct digits b/w 1000 & 9999?

Combinations: n, r : non-negative integers.

A r-combination of a set of n distinct elements means a subset of size r

from the set.

$\binom{n}{r}$, ${}^nC_r$, $C(n,r)$: # r-combinations

of a set of n elems.

$$\binom{n}{r} = \begin{cases} n & , \quad r = 1 \\ 1 & , \quad r = 0, n \\ 0 & , \quad r > n \end{cases}$$

The procedure for constructing a r-permutation

is as follows.

S_1 : Choose a r-subset from the set of n elems. $\quad \binom{n}{r}$

S_2 : Order the r elements. $\quad r!$

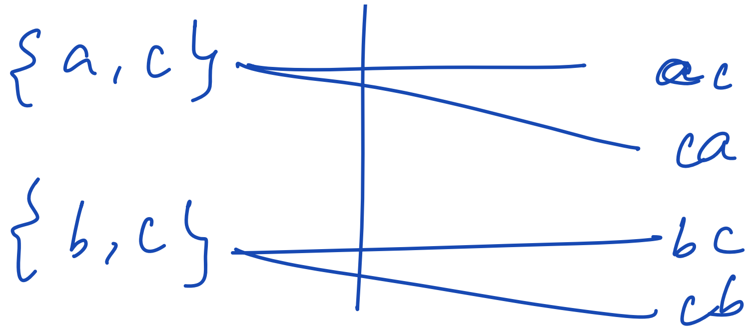
By the MR,

$$P(n,r) = \binom{n}{r} \cdot r!$$

$$\therefore \boxed{\binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{r!\,(n-r)!}}$$

$S = \{a, b, c\}$, $r = 2$

r-subsets	r-permutations
$\{a, b\}$	ab
	ba



Ex: 8 women, 6 men.

committees containing 3 women & 2 men.

What if two of the men are feuding & refuse to serve on the committee together?

Soln The proc. of constr. such a committee is as follows.

S_1: Choose the 3 women.	$\binom{8}{3}$
S_2: ———— 2 men.	$\binom{6}{2}$

By the MR, # committees $= \binom{8}{3}\binom{6}{2}$.

Let a & b be the two feuding men.

S' : set of all committees in which a & b do not serve together.

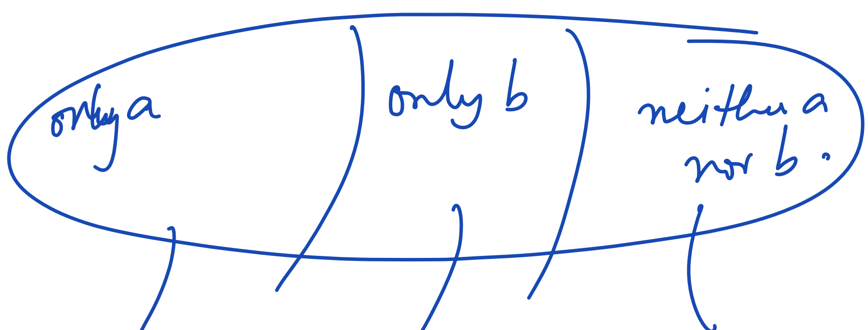
S'' : set of all committees in which a & b serve together.

S : all committees.

S' & S'' partition S.

$$|S'| = |S| - |S''|$$

$$= \binom{8}{3}\binom{6}{2} - \binom{8}{3}$$



$$\binom{4}{1} \quad \binom{4}{1} \quad \binom{4}{2}$$

Ex: 15 students enrolled in the course

exactly 12 attend on a given day.

25 distinct seats

seatings possible?

Soln: The proc. of constr. a classroom seating is as follows:

S_1: Choose the 12 students. $\binom{15}{12}$

S_2: Choose 12 seats out of 25. $\binom{25}{12}$

S_3: Seat the 12 students $12!$

S_{2a} find a seat for 1^{st} student 25

2^{nd} — 24

3 — — .

By the MR, our ans $= \binom{15}{12} \underbrace{\binom{25}{12} 12!}_{P(25,12)}$

C(25,12)

Ex: # 8-letter strings constructed using 26 letters of the alphabet that contain exactly 3, 4, or 5 vowels?

No restriction on # times a letter is used.

What if a letter can be used at most once?

Soln:

S

exactly 3 vowels | exactly 4 vowels | exactly 5 vowels

$\binom{8}{3} 5^3 21^5$

_ _ _ _ _ _ _ _

1 2 3 4 5 6 7 8

The proc. of constructing 8-letter

3 vowels.

S_1 : find the locⁿ for vowels.

S_2 : Identify the 3 vowels.
S_{2a} : find the loc. for

S_3 : Identify the 5 consonants.

ways to do S_1 : $\binom{8}{3}$

——— S_2 : 5^3

S_3 : 21^5

Multisets

$$M = \{2 \cdot a,\ 3 \cdot b\}$$

perm of elems in M?

$$M = \{2 \cdot a,\ 1\, b\} \qquad \{a_1, a_2, b\}$$

$a\,a\,b$	$a_1\,a_2\,b$
	$a_2\,a_1\,b$
$a\,b\,a$	$a_1\,b\,a_2$
	$a_2\,b\,a_1$
$b\,a\,a$	$b\,a_1\,a_2$
	$b\,a_2\,a_1$

$M = \{n_1 \cdot T_1, n_2 \cdot T_2, \ldots\ldots, n_k \cdot T_k\}$

$n_1 + n_2 + \cdots + n_k = n.$

\# permutations of elements in M.

The proc. of constr. a perm. of elems in M is as follows:

S_1: Choose the n_1 positions for T_1 objects $\binom{n}{n_1}$

S_2: ——— n_2 ——— T_2 — $\binom{n-n_1}{n_2}$

⋮

S_k ——— n_k ——— T_k obj-.

$\underline{\ \ }_1 \quad \underline{\ \ }_2 \quad \underline{\ \ }_3 \quad \cdots \quad \underline{\ \ }_n$

\# permutations of the elements in M

$$= \binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots$$

$$= \frac{n!}{n_1!\,\cancel{(n-n_1)!}} \cdot \frac{\cancel{(n-n_1)!}}{n_2!\,\cancel{(n-n_1-n_2)!}} \cdot \frac{\cancel{(n-n_1-n_2)!}}{n_3!\,\cancel{(n-n_1-n_2-n_3)!}} \cdots$$

$$= \boxed{\frac{n!}{n_1!\,n_2!\cdots n_k!}}$$