No office hours today.

Ex: # ways to order 26 letters of the alphabet so that no two vowels a, e, i, o, u appear consecutively?

Soln: The procedure of constructing a permutation of the letters s.t. no two vowels occur consecutively is as follows:

S_1: Arrange the consonants.

S_2: Arrange the five vowels in the 22 spaces b/w & around the consonants.

S_{2a}: Choose a posn for a $\longrightarrow$ 22

S_{26} : ______ e → 21
i → 26

\# ways to do Step 1 : $21!$

\# ways to do Step 2 : $P(22,5)$

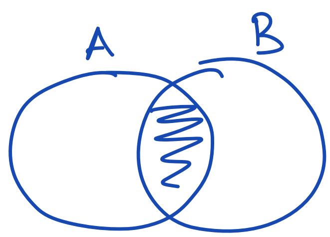
By the MR, total \# permutations $= \underline{21! \cdot P(22,5)}$.

_ m _ l _ b _ d _ p _ q _ r _ . . .

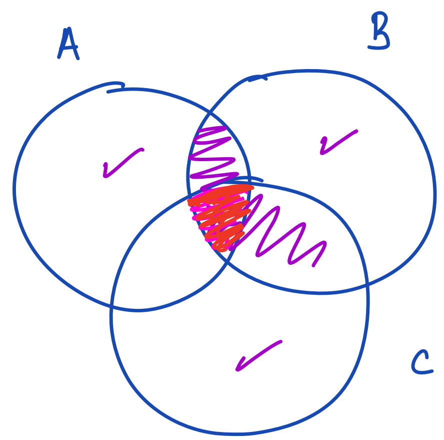
19-20 0-1 3-4

a e i o u

Inclusion-Exclusion



$$|A \cup B| = |A| + |B| - |A \cap B|$$



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

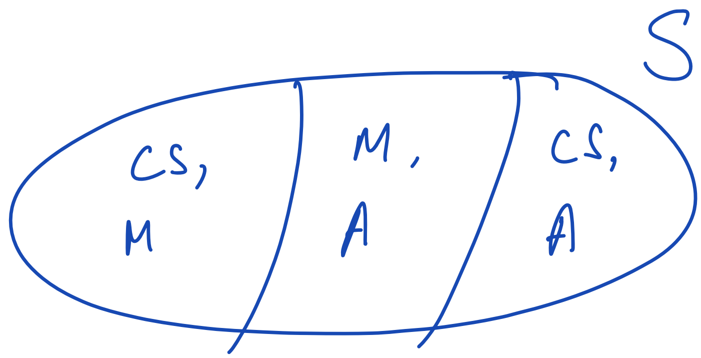
Addition Rule.

If sets A & B are disjoint then

$$|A \cup B| = |A| + |B|.$$

Ex: # ways to select two books from different subjects among five distinct CS books, three distinct Math books & two distinct Art books?

Soln:



Let S be the set of all pairs of books chosen from different subjects. S can be <u>partitioned</u> into S_1, S_2, S_3

where

S_1 : set of pairs of books with one CS & one Math

S_2 : ——————————————— CS & one Art

S_3 : ——————————————— M, one Art.

$|S_1| = 15$, $|S_2| = 10$, $|S_3| = 6$.

$$|S| = |S_1| + |S_2| + |S_3|$$

$$= 31.$$

Ex: # int b/w 1 & 1000 that are multiples of 3 or multiples of 5?

Soln: Let S_3 be the set of int b/w 1 & 1000 that are multiples of 3.

S_5 : set of int b/w 1 & 1000 that are multiples of 5.

$|S_3| = 333$, $|S_5| = 200$

Clearly, our ans $= |S_3| + |S_5| = 533$.

S_3 & S_5 are not disjoint.

$|S_3 \cap S_5| = |S_{15}| = 66$

$15 \quad \ldots\ldots \quad 990$

$15 \cdot 1 \quad\quad 15 \cdot 66$

$\therefore$ Our answer: $333 + 200 - 66$

Ex: # 4-letter strings that contain the letter x.

atleast one x.

Soln: The proc. of constructing a 4-letter string that contains x is as follows.

S_1: Choose a posn for x.

~~S_2: Choose a letter for each of the other posns.~~

Bogus!

~~By the MP, # such strings $= 4 \cdot 26^3$.~~

alphabet $= \{x, a\}$

$$\left.\begin{array}{cc} \underline{x} & \underline{a} \\ \\ \underline{x} & \underline{x} \\ \\ \underline{x} & \underline{x} \\ \\ \underline{a} & x \end{array}\right\} 4$$

xa

ax

xx

Let S be the set of all 4-letter strings.

Let S_1 be the set of all 4-letter strings that contain x.

Let S_2 be the set of all 4-letter strings

that do not contain x.

S_1 & S_2 partition S.

$\therefore \quad |S| = |S_1| + |S_2|$

$$|S_1| = |S| - |S_2|$$

$$= 26^4 - 25^4 \checkmark.$$

Ex: # 4-digit even #s that have

distinct digits?

Combinations.

n, r : non-negative integers.

A <u>r-combination</u> of a set of n distinct elements is a subset of size r.

$\binom{n}{r}$, ${}^nC_r$, $C(n,r)$: # r-subsets of a set of n distinct elements.

$$\binom{n}{r} = \begin{cases} n & , \ r = 1 \\ 1 & , \ r = 0, \ r = n \\ 0 & , \ r > n \end{cases}$$

The procedure to construct a r-permutation is as follows.

S_1 : Choose the r elements	$\binom{n}{r}$
S_2 : Order them	$r!$

By the MR,

$$P(n,r) = \binom{n}{r} \cdot r!$$

$$\therefore \binom{n}{r} = \frac{P(n,r)}{r!}$$

$$\boxed{\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}}.$$

Ex: # 11-member teams from a pool of 14 players?

Soln: $\binom{14}{11}$.

Ex: 8 women, 6 men.

different committees consisting of 3 women & 2 men? What if two of the men are feuding & refuse to be in the committee together?

Soln: The proc. of constr. such a committee is as follows.

S_1: Choose the 3 women. $\binom{8}{3}$

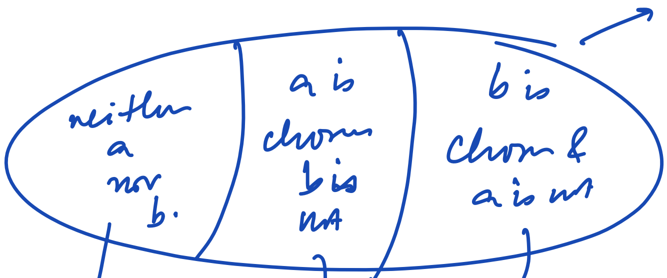
S_2: ———— 2 men $\binom{6}{2}$

ans : $\binom{8}{3}\binom{6}{2}$.

In the case of feuding men,

ans : $\binom{8}{3}\binom{6}{2} - \binom{8}{3}\binom{2}{2}$

$$\binom{8}{3}\left(\binom{6}{2}-1\right)$$



$\binom{4}{2}$ $\quad$ $\cancel{\binom{5}{1}}$ $\quad$ $\cancel{\binom{5}{1}}$.

$\binom{4}{1}$ $\quad$ $\binom{4}{1}$

Ex: - 15 students in a course.

- exactly 12 attend on any given day.
- classroom has 25 distinct seats.
- # different classroom seatings possible?

Soln: The proc. of constr. a classroom seating is as follows.

S_1: Choose the 12 students.

Seat the 12 students.

S_2: ~~Choose the 12 seats.~~

$$\# \text{ seatings} = \cancel{\binom{15}{2}} \cdot \binom{25}{12}.$$

$$= \binom{15}{12} \cdot P(25, 12).$$

S_1: Choose the 12 students.

S_2 : Choose the 12 seats.

S_3 : Seat the 1^{st} student.

S_4 : ———— 2^{nd} – –

⋮

12^{th} st

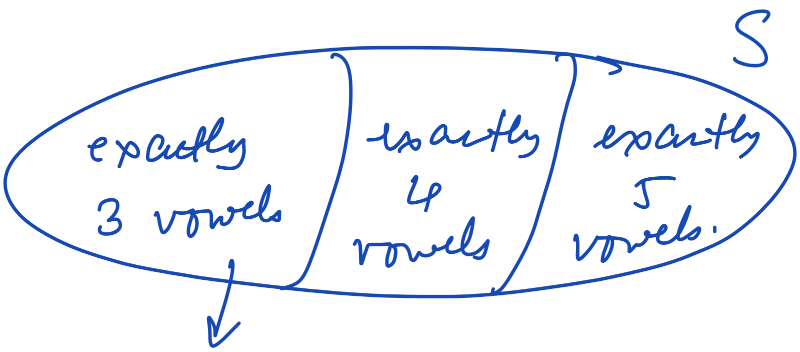
$$\binom{15}{12} \cdot \binom{25}{12} \cdot 12 \cdot 11 \cdot \cdots \cdot 1 .$$

$$\binom{15}{12} \binom{25}{12} \cdot 12!$$

$$= \binom{15}{12} \cdot P(25, 12) .$$

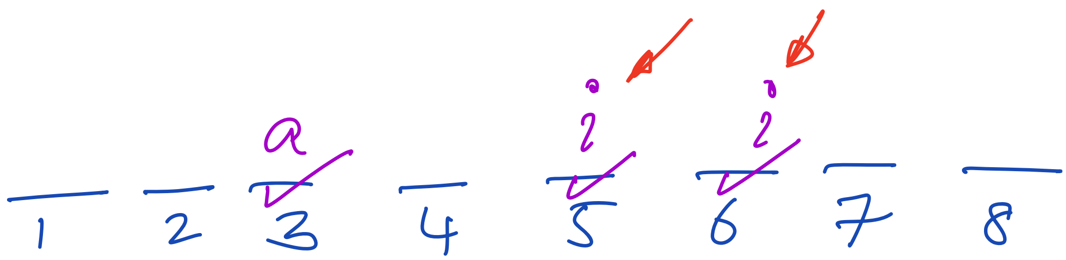
Ex: # 8-letter strings that can be constructed using 26 letters of the alphabet if each string contains 3, 4 or 5 vowels?

Soln:



8-letter strings with exactly 3 vowels.

(no restriction on # occurrences of each letter).



answer : $\cancel{\underbrace{\binom{8}{3} \cdot P(5,3) \cdot P(21,5)}} \cdot 5^3$.

$\binom{8}{3} \cdot 5^3 \cdot 21^5$.

all letters are distinct?
no letter can be repeated?

Permutations of multisets:

$M = \{3 \cdot a, 2 \cdot b\}$

permutations of elements in M?

$$M = \{n_1 \cdot T_1,\ n_2 \cdot T_2,\ \dots,\ n_k T_k\}$$

s.t. $n_1 + n_2 + \dots + n_k = n$.

permutations of elems in M ?

<u>Example</u> : $M = \{2 \cdot a,\ 1 \cdot b\}$

$\{2 \cdot \underline{a},\ 1 b\}$	$\{a_1, a_2, b\}$
$a\,a\,b$	$a_1\,a_2\,b$ $a_2\,a_1\,b$
$a\,b\,a$	$a_1\,b\,a_2$ $a_2\,b\,a_1$
$b\,a\,a$	$b\,a_1\,a_2$ $b\,a_2\,a_1$

~~The~~ procedure of constr. a permutation of elems in M is as follows:

S_1 : Chose the n_1 posns for T_1 $\binom{n}{n_1}$

S_{1a} : Place T_1 obj.

S_2 : ________ n_2 T_2 $\binom{n-n_1}{n_2}$

S_{2a} : Place T_2 obj.

$\vdots$

n_k ____ T_k.

$$\#\text{ permutations} = \binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots$$

$$= \frac{n!}{n_1!\,\cancel{(n-n_1)!}} \cdot \frac{\cancel{(n-n_1)!}}{n_2!\,\cancel{(n-n_1-n_2)!}} \cdot \frac{\cancel{(n-n_1-n_2)!}}{n_3!\,\cancel{(n-n_1-n_2-n_3)!}} \cdots$$

$$= \boxed{\frac{n!}{n_1!\,n_2!\,n_3!\cdots n_k!}}$$