CIS 1600 Assigned: October 21, 2024

Homework 9T

Due: 11:59PM EDT, October 23, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, October 23, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Ed.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [12 pts] Tree-ck Or Tree-t

The two Kevins are planning to go trick or treating this year and want to create a map of their neighborhood G made up of houses with roads between them. Each road will go between two houses and there is at most one road between each pair of houses. They each make a map by drawing undirected roads between houses, hitting every house. To minimize walking time, they have their maps be minimally-connected. In other words, the two maps are both spanning trees of G.

If Kevin Han's map forms the spanning tree $T = (V, E)$ and Kevin Song's map forms the spanning tree $T' = (V, E')$, prove that for any road $e \in E \setminus E'$, there is a road $e' \in E' \setminus E$ such that $(T - e) + e'$ and $(T' - e') + e$ are also spanning trees of G.

2. [8 pts] Don't Knock on Will's Door

Finding his life to be too boring, Will has decided to live stochastically. So, this year for Halloween, if a trick-or-treater knocks on his door, he will flip a coin to decide what to give them: A MrBeast Feastables bar for heads and a Prime Hydration drink for tails.

8 trick-or-treaters are lined up at Will's door and he hands out items accordingly. Define the events A, B, C as follows:

A: event that his first toss results in Tails.
B: event that his fourth toss results in Heads.
C: event that there are a total of exactly 4 Heads.

Help Will understand what he is getting himself into by answering the following questions with proper justification. Only answers with an appropriate sample space Ω will receive full credit.

(a) Are events A and B independent?

(b) Are events A and C independent?

(c) Are events B and C independent?

(d) Are events A, B, and C mutually independent?

3. [10 pts] Things are Getting Pumpkin Spice-y

Fall-ika and Pa-trick-or-treat Dugan both love Starbucks Pumpkin Spice Lattes; however, there's only one left. After debating for a while, neither of them is willing to give up this delicious fall drink. To determine who gets the drink, they roll four fair 6-sided dice, and the person who correctly predicts the largest integer that appears on any of the four fair-sided dice will be declared the winner. Compute the expected value of the largest integer that appears on any of the four dice, and tell your favorite TA so they will have a better chance of gulping down this fall special!