Homework 8T

This homework is due electronically on Gradescope at 11:59PM EDT, October 16, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:
   - 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Ed.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. **[9 pts] Sussy**

   Suzzy, as a dedicated crewmate, is rushing to complete her final task aboard the spaceship
   before the Impostor strikes again! Her task is located far away in Electrical, but there's a
   problem—Suzzy's movement system is malfunctioning due to a sabotaged navigation system.

   The malfunction means that each time Suzzy attempts to move, there's a chance with probability
   p that she advances 1 unit closer to Electrical to finish her task and a probability $1 - p$ that
   Suzzy's movement is blocked, where she stays in place, unable to make progress.

   (a) What is the probability that after making two attempts to move, Suzzy will be 1 unit closer
   to Electrical than she was when she started?

   (b) What is the probability that after making three attempts to move, Suzzy will be 2 units
   closer to Electrical than she was when she started?

   (c) Given that Suzzy is 2 units closer to Electrical after making three attempts to move, what
   is the probability that after her first attempt to move, Suzzy ended up 1 unit closer to
   Electrical?

2. **[9 pts] E-Rico Zhao**

   Recitation just ended, and you know what that means—it's time for the TAs to brawl it out in an
   epic round of Solo Showdown! Eric decides that his best bet is to camp out in the bushes while
   stealthily collecting as many powercubes as possible. Then, at the last minute, he'll overwhelm
   the competition with his insurmountable power!

   Before the round starts, he quickly surveys the map to plan the most strategic route for collecting
   powercubes. He notices that there are $2k$ bushes and k roads, each connecting exactly two
   bushes. Assume that k is a positive integer, and that all roads are bidirectional. Help Eric Zhao
   prove that if each bush has at least one road connecting to it, then there are exactly k disjoint
   groups of bushes, where each bush in a group is reachable from all other bushes in that group
   by some sequence of roads and each bush can only reach the bushes in the same group.

3. **[12 pts] A Blast to the Past**

   Jan is playing a Candy Crush knockoff right before lecture. In this game, there's a 3×3 grid
   with each square filled with a red or blue candy. On each turn, the player can tap exactly
   one candy to destroy it. When a candy is destroyed, it will destroy all other candies directly
   adjacent to it (not diagonally adjacent) with the same color. Each destroyed candy will then
   destroy all adjacent candies of the same color, causing a chain reaction.

   The board is initially filled in uniformly at random (i.e. every grid cell has an independent and
   uniform $\frac{1}{2}$ chance of being either a red candy or blue candy). Max bets Jan that he won't be
   able to clear a 2×2 block of red candies on one turn. Help Jan prove Max wrong and find the

probability that the starting board contains a 2×2 block of red candies that can be cleared by tapping on a red candy!