CIS 1600 Assigned: September 30, 2024

Homework 6T

Due: 11:59PM EDT, October 2, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, October 2, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: You may not collaborate with anyone via any means.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [6 pts] Om Nom Nom!

Answer the following questions. You may express your answer as a percentage, decimal or fraction. **For this question only, you do not need to show your work. Only your final answer will be graded.**

For questions (a), (b), (c), find the probabilities of the events described based on the following situation: The menu at Burger King lists out n distinct Whoppers labeled $1, 2, \ldots, n$. Darren "the biggest munch" Chen is visiting Burger King for lunch, and picks a Whopper to eat uniformly at random. Still hungry, he returns for dinner and again picks a Whopper to eat uniformly at random. Note that Darren can eat the same Whopper for lunch and dinner should he wish. Question (d) is based on a similar situation (read the problem statement below for details).

(a) The first Whopper chosen is number 1 and the second Whopper chosen is number n.

(b) The numbers of the two Whoppers are consecutive integers with the first Whopper's number being one less than the second Whopper's number.

(c) The second Whopper chosen has a higher number than the first Whopper chosen.

(d) Darren sees a promotion at Burger King that offers a free meal to the first person that eats two different Whoppers. Wanting to be crowned the Burger King, he does not want to eat the same Whopper for lunch and dinner. Now, recalculate (a), (b), and (c), assuming this time that Darren can never eat the same Whopper for lunch and dinner.

2. [8 pts] Wing Stop

Winston wanted to get Buffalo Wild Wings for lunch today! He bought a tray of $n \geq 1$ wings for himself and went over to the counter to pick sauces. However, he trips in a rush to eat the wings and notices the sauces flowing all over the tray. There are $m \geq 0$ sauce pathways that form between pairs of wings and are bidirectional. Each sauce pathway runs between exactly two wings and there is at most one sauce pathway between two wings. Prove that $n \leq n^2 - 2m$.

3. [8 pts] McDonald's Gatekeeping

With his latest order, Patrick just earned 4500 McDonald's rewards points. To treat himself, he goes right back in to redeem a Filet-O-Fish with all his points. However, before entering, he is stopped by Grace the McDonald's Troll. She won't let him pass unless he wins in the following game.

Patrick will select a card uniformly at random from a standard deck, and then without putting it back, he will select a second card uniformly at random from the remaining cards. He will win if both cards have rank no higher than 10, and at least one of the cards is red. Note that an ace is considered the highest rank.

What is the probability Patrick will be able to enter the McDonald's and order a delectable Filet-O-Fish?

4. [8 pts] The Burger King

Andrew, the Burger King, wants to launch a secret new menu item, but he's worried that Ronald McDonald, who just opened his restaurant on Penn's campus, might find out. To spread the news strategically, Andrew tells all of the franchises he is in direct contact with about the menu item first. Each franchise then shares the secret with their direct contacts. Suppose there are $n \geq 3$ franchises, and each franchise (including Andrew's Burger King) contacts at least $\frac{n}{2}$ other franchises through supply chains. Prove that any two franchises either share a direct connection or there exists a mutual partner franchise that is in direct contact with both.