CIS 1600 Assigned: September 16, 2024

Homework 4T

Due: 11:59PM EDT, September 18, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, September 18, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: All solutions are required to be typeset in LaTeX.

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: You may not collaborate with anyone via any means.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [10 pts] SpongeBob RectanglePants

One morning, SpongeBob suddenly woke up with rectangle pants thanks to the nefarious plans of Davidon, Plankton's cousin! Davidon threatens that if SpongeBob doesn't prove that the below inequality holds for $n > 1$, he will never get his square pants back.

Help SpongeBob stop Davidon's schemes by proving that for all integers $n > 1$,

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

2. [10 pts] Fairly Counting Parents

Timmy Turner, Wanda, and Cosmo are preparing to go on their next adventure! To finish the adventure, they need to make exactly 22 wishes. Timmy, Wanda, and Cosmo are each thinking of wishes (they can make zero wishes and cannot make fractions of wishes). However, Vickytor, Timmy's evil babysitter, has special powers! Vickytor can choose to contribute anywhere between -2 and 9 (inclusive) magic wishes to help (or hinder) their total. Let c, w, t be the natural number of wishes that they make and v be the integer number of wishes Vickytor makes. How many different ways can Cosmo (c), Wanda (w), Timmy (t), and Vickytor (v) make wishes such that $c + w + t + v = 22$?

3. [10 pts] Tubby Troubles

In a secret, un-aired episode of Teletubbies, Tinky-Winky and Laa-Laa get locked out of the Tubbytronic Superdome. In order to get back in, they have to provide a combinatorial proof for the following identity:

For any $n \geq 1$,

$$\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2$$

Help Tinky-Winky and Laa-Laa come up with a combinatorial proof so they can get back home before before Tubby Bye-Bye time!