CIS 1600

Homework 3T

Assigned: September 9, 2023

Due: 11:59PM EDT, September 11, 2023

This homework is due electronically on Gradescope at 11:59PM EDT, September 11, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Normally, we require all solutions to be typeset in LaTeX. We have provided a LaTeX primer video on Piazza and on the course website under the 'resources' tab, and have provided a template, should you choose to use LaTeX. However, LaTeX is not strictly required **for this first assignment only.**

B. Standard Deductions:
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: You may not collaborate with anyone via any means.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [9 pts] D(arren) R(eviews) L(eaves)

Legendary biologist Charles Darren is studying the flora in the jungle outside of DRL when Darren notices a fascinating pattern in the leaves. He notices that every plant of the species *computatrum scientia MDC* has a specific relationship between the number of margins and veins on the leaf. For a given leaf with any positive integer a veins, the leaf will have c margins where $c \in \mathbb{N}$ and $a = 14c^2$. However, he also comes up with another revelation: $\sqrt{a}$, the length of the midrib, is always irrational. Help Darren prove that this is the case.

2. [9 pts] The Jungle of Brotherly Love

The King of the Jungle, David Mu-*Fu*-sa, is ready to peacefully retire and become a stay-at-home dad, after a work-related injury. As he wants his successor to be sufficiently bureacratically skilled, he sorts his army of howler monkeys based off their favorite trees: rubber trees (R), brazil nut trees (B), spruce trees (S), and fir trees (F). On the other hand, "Scar"-ya Singhi has always wanted to be king, and sees an opportune moment. Help him complete these proofs! Note that these monkeys are indecisive and change their minds often, so between each subproblem, the monkeys in each of the sets R, B, S, and F may change!

(a) Suppose that R, B, and S are sets with $R \cap B \cap S = \emptyset$. Prove or disprove:

$$|R \cup B \cup S| = |R| + |B| + |S|$$

(b) Let R, B, S, and F be arbitrary sets. Prove or disprove:

$$(R \cap S) \cup (B \cap F) \subseteq (R \cup B) \cap (S \cup F)$$

(c) Let R and B be arbitrary sets. Prove or disprove:

$$\mathcal{P}(R) \cup \mathcal{P}(B) = \mathcal{P}(R \cup B)$$

3. [12 pts] Monkey Kidnapping

While exploring the jungle, Andrew has been captured by monkeys! The monkeys demand that he determine the validity of a few proofs in order to be set free. Unfortunately, it's been a while since Andrew has done any proofs, so he asks you for help.

For each of the "proofs" below, say whether the proof is valid or invalid. If it is invalid, indicate clearly as to where the logical error in the proof lies, and justify why this is a logical error. If the proof is valid, you may simply say so. Just stating that the claim is false will not be awarded credit.

(a) **Claim:** All natural numbers are divisible by 143.

Proof: Suppose, for the sake of contradiction, the statement were false. Let C be the set of counterexamples, i.e., $C = \{c \in \mathbb{N} \mid c \text{ is not divisible by } 143\}$. The supposition that the statement is false means that $C \neq \emptyset$. Since C is a nonempty set of natural numbers, it contains at least element z.

Note that $0 \notin C$ because 0 is divisible by 143. So $z \neq 0$. Now consider $z - 143$. Since $z - 143 < z$ (and z is the smallest counterexample) then $z - 143$ is not a counterexample to the original statement and is therefore not in C. Therefore $z - 143$ is divisible by 143; that is, there is an integer a such that $z - 143 = 143a$. So $z = 143a + 143 = 143(a + 1)$ and z is divisible by 143, contradicting $z \in C$.

(b) **Claim:** For all natural numbers n, if $2n + 1$ is a multiple of 3, then $n^2 + 1$ is a multiple of 3.

Proof: We will prove the contrapositive. Assume $2n + 1$ is not a multiple of 3.

- If $n = 3a$, for $a \in \mathbb{N}$, then $n^2 + 1 = 9a^2 + 1$ is not a multiple of 3.
- If $n = 3a + 1$ for $a \in \mathbb{N}$, then $(2n + 1) = 6a + 3$ is a multiple of 3, so the original claim holds, as false implies everything.
- If $n = 3a + 2$ for $a \in \mathbb{N}$, then $n^2 + 1 = 9a^2 + 12a + 5$ is not a multiple of 3.

In all cases, we have concluded $n^2 + 1$ is not a multiple of 3, so we have proved the claim.

(c) **Claim:** If $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$, then $4xy^3$ has a different parity than x.

Proof: Assume, without loss of generality, that x is odd. By definition of an odd integer, $x = 2k + 1$, where $k \in \mathbb{Z}$. Thus:

$$4xy^3 = (2k + 1)(4y^3) = 8ky^3 + 4y^3 = 2(4ky^3 + 2y^3)$$

Since $4ky^3 + 2y^3 \in \mathbb{Z}$, $4xy^3$ is even and hence has a different parity than x.