CIS 1600

Homework 3H

Assigned: September 11, 2023

Due: 11:59PM EDT, September 16, 2023

This homework is due electronically on Gradescope at 11:59PM EDT, September 16, 2023. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Normally, we require all solutions to be typeset in LaTeX. We have provided a LaTeX primer video on Piazza and on the course website under the 'resources' tab, and have provided a template, should you choose to use LaTeX. However, LaTeX is not strictly required **for this first assignment only.**

B. Standard Deductions:
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: You may not collaborate with anyone via any means.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [27 pts] Shelter by Induction

Jungle expert Jonathan is in a classic jungle pinch. He has no shelter. This wouldn't normally be a problem with Jonathan's skills, but his mind has been too preoccupied thinking about 1600 math problems to focus on the task. He thinks to himself: "Gaahh!! If only I could formulate proofs for the following 3 induction problems, then my brain would be free to build this shelter in a flash!" Create proofs for the following induction problems to help Jonathan focus on his survival.

(a) Prove that for all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$$

(b) Prove by induction that for $\forall n \in \mathbb{Z}^+$,

$$2^n + 3^n < 6^n.$$

(c) Prove by induction that $9^n - 5^n - 4^n$ is divisible by 20 for all $n \in \mathbb{Z}^+$.

2. [10 pts] The Serpent Count

Luna loves taking walks in the jungle. On one of her walks, Luna gets bored and decides to count the number of snakes on the way. As she walks into the jungle, she counts a total of $k\binom{n}{k}$ snakes where n and k are positive integers such that $n \geq k$. Then she walks back out the jungle counting the snakes again, this time with a different method and counts a total of $n\binom{n-1}{k-1}$ snakes. Help explain why she counted the same number of snakes each time by giving a combinatorial proof for the following identity:

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

where $k, n \in \mathbb{N}, 1 \leq k \leq n$.

3. [12 pts] Cindy's Court of Creatures

Cindy is hiking through the Amazon jungle and taking pictures of all the cool animals she spots to show the other CIS 1600 TAs. Suddenly, she realizes that it would be an even better idea to befriend the animals and bring them with her back to Philly! Because she has limited space in her carry on, she decides to befriend exactly $r \geq 1$ individual animals. The Amazon jungle contains $k \geq 1$ species of animals, with an infinite number of each species besides the jaguar, of which there is only 1 because it is the apex predator. How many ways can Cindy create her animal friend group? Note that animals of the same species are indistinguishable and the jaguar counts as one of the k species.

4. [10 pts] Jungle Jumble

Darren the Explorer is is about to enter a perfectly square jungle to find his pet crocodile.

Luckily, the walkways between the trees form a perfect 7×7 coordinate grid. The North, South, East, and West cardinal directions correspond to the up, down, right, and left directions on his map.

Looking at his map, he notes that he's currently at the bottom-left of the grid corresponding to $(0, 0)$, and the last known sighting of his crocodile is at the top-right of the grid corresponding to $(7, 7)$. Darren starts planning his route to the crocodile, but he gets dizzy if he takes too many turns. If he can only travel in the North or East directions, how many paths along the walkways in between trees can he take such that he only turns exactly four times?

5. [11 pts] The T-imaster's Toils

As Maxwell T-i, the T-imaster is wandering around the jungle, Maxwell finds species of red flowers, green flowers, and blue flowers. Maxwell thinks of a way to make a special tea. In order to make this drink, Maxwell must perform the steps in the following action: Maxwell will crush the red flower into a bottle, then Maxwell will pull the green flower's petals and place them into the bottle, and finally Maxwell will place the entire blue flower on top of the other flowers. Maxwell has also placed the flowers corresponding with each drink next to its respective bottle. In other words, the flowers are labelled from $1 - 36$.

For each of the 36 herbal drinks, Maxwell must perform these three steps in this exact chronological order or else the drink will mix improperly and lose its taste. However, Maxwell may choose to work on and/or finish another drink before finishing the drink Maxwell is currently on. In how may distinct orders can Maxwell create all 36 special teas?

For example, to create herbal drink 1 and herbal drink 2, Maxwell could process:

(a) Red flower for drink 1

(b) Red flower for drink 2

(c) Green flower for drink 2

(d) Blue flower for drink 2

(e) Green flower for drink 1

(f) Blue flower for drink 1