

Homework 2T

Due: 11:59PM EDT, September 04, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, September 04, 2024. To receive full credit all your answers should be carefully justified. **Additionally, make sure to fill out the Gradescope Policy Quiz!**

Please make note of the following:

- A. L^AT_EX:** Normally, we require all solutions to be typeset in L^AT_EX. We have provided a L^AT_EX primer video on Piazza and on the course website under the ‘resources’ tab, and have provided a template, should you choose to use L^AT_EX. However, L^AT_EX is not strictly required **for this first assignment only**.
- B. Standard Deductions:**
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- C. Solutions:** Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.
- D. Collaboration:** You may not collaborate with anyone via any means.
- E. Citations:** All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.
- F. Outside Resources:** Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.
- G. Late Policy:** We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
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1. [9 pts] Platt Performance

Chukka is trying to buy a ticket to attend a Penn Band show in the Platt Performing Arts House, but he realized that he didn't bring any money. After much pleading, David agrees to give him a free ticket if he is able to answer three CIS 1600 questions. Help Chukka with the following proofs so he can watch David and the Penn Band put on an awesome performance!

(a) Let $A = \{n \in \mathbb{N} \mid n = 7k+4, \text{ for some } k \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} \mid n = 21k + 4, \text{ for some } k \in \mathbb{N}\}$.

Prove that $B \subset A$.

(b) Let $A = \{n \in \mathbb{Z} \mid n = 4 - k, \text{ for some } k \in \mathbb{N} \text{ and } 5|k\}$ and $C = \{m \in \mathbb{Z} \mid m = 24 - 5k \text{ for some } k \in \mathbb{N} \text{ and } k \geq 4\}$. Prove that $A = C$.

(c) Let $A = \{n \in \mathbb{N} \mid n = 6k-13, \text{ for some } k \in \mathbb{N}\}$ and $B = \{m \in \mathbb{N} \mid m = 5k + 14, \text{ for some } k \in \mathbb{Z}^+\}$.

Prove that $A \neq B$.

2. [5 pts] Amazommons Problems

Luke walks to Amazommons from the high rises. Unfortunately, he forgot the password to his Amazon account! Amazon@Penn tells him that he can get his package if can give a valid proof for $\sqrt{6} + \sqrt{7} < \sqrt{26}$.

Luke confidently responds with the following:

“Squaring both sides of $\sqrt{6} + \sqrt{7} < \sqrt{26}$ gives $13 + 2\sqrt{42} < 26$, which further implies $2\sqrt{42} < 13$. Squaring both sides gives $168 < 169$, which is true.”

Help the Amazommons worker verify Luke's solution. If it is valid, give a brief justification why (a couple of lines will suffice). If not, explain why the proof is invalid and provide a correct proof.

3. [10 pts] Book Thief!

Thomas is leaving Van Pelt for the first time before being suddenly stopped by an enraged security guard. She tells him he must prove or disprove the following statements lest he be considered a book thief.

(a) \forall primes p , either $p + 10$ is a prime or $p + 11$ is a prime.

(b) $\forall \alpha, \beta \in \mathbb{R}^+, \lfloor \alpha + \beta \rfloor = \lfloor \alpha \rfloor + \lfloor \beta \rfloor$.

(c) $\forall \epsilon \in \mathbb{N}, (\epsilon + 1)^3 - 5\epsilon^2 + 6$ is odd.

(d) $\forall x, y, z \in \mathbb{Z}, (\gcd(x, y) = 1) \wedge (\gcd(y, z) = 1) \implies \gcd(x, z) = 1$.

(e) $\forall q, r, s \in \mathbb{Z}$, if $q - r$ is even and $r - s$ is odd, then $q - s$ is odd.

Help Thomas clear his name via the power of logical deduction.

4. [6 pts] Levine Lobby Luncheon

As Yinqi leaves 1600 lecture, she realizes she is famished. Luckily, she happens upon Sophia setting up for the CIS department welcome-back luncheon in Levine lobby. Sophia offers her a deal: if Yinqi can help order the trays of food on the table, she can take a plate for herself. There are 59 trays of distinct dishes, 24 of which are vegan and non-vegan versions of the same dish (12 pairs). Each vegan/non-vegan pair should be directly next to each other. If trays are placed in a single line down the table, how many ways can Yinqi arrange the dishes?