CIS 1600 Assigned: September 4, 2024

Homework 2H

Due: 11:59PM EDT, September 9, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, September 9, 2024. To receive full credit all your answers should be carefully justified. **Additionally, make sure to fill out the Gradescope Policy Quiz!**

Please make note of the following:

A. LaTeX: Normally, we require all solutions to be typeset in LaTeX. We have provided a LaTeX primer video on Piazza and on the course website under the 'resources' tab, and have provided a template, should you choose to use LaTeX. However, LaTeX is not strictly required **for this first assignment only.**

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: You may not collaborate with anyone via any means.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [8 pts] Harrison Home <3

Olivia is moving into a dorm room on the first floor of Harrison after a year of living in a tiny freshman dorm. She wants to make the best use of all the space in her room which is a cube. She wants to put two dressers of equal length next to each other without any space and hopes that the side length is even. She makes an observation that "if the volume of my room is an even integer and a perfect cube, the side length of the room would be even too!" Can you help Olivia prove her claim? Recall that the volume of a cube with side length s is s^3.

2. [10 pts] JonaPHINS

Jonathan is leading his PHINS group across campus when he hears that Andrew got lost in the basement of DRL. To ensure no one else gets lost in the depths of DRL, Jonathan insists that his students travel using the "buddy" system, meaning that his students must pair up before entering DRL. Jonathan's group happens to consist of 864 **distinct** students. How many ways can Jonathan pair up students in his group to search for Andrew? (Note that Andrew is not part of the 864 students, the order of the students within a pair does not matter, and the ordering of the pairs doesn't matter either).

3. [12 pts] The PennCard Puzzle

Sid, Quad's newest and brightest RA, has bought 8 distinguishable replacement PennCards for the freshmen on his floor in case they lose theirs. After their first chaotic frat party, 3 (distinguishable) freshmen on Sid's floor return to the Quad having lost their PennCards. Since Sid's wallet is already too heavy from his recent summer internship, he wants to get rid of all 8 PennCards. How many ways can Sid give away all 8 PennCards to the 3 freshmen, keeping in mind that they all must receive at least one so they can get back into their rooms?

Note: Any freshman can receive any one of the PennCards.

4. [10 pts] This'll Make an Awesome Dorm Decoration

It's move-in day, and Grace is admiring the beautiful facade of her new dorm, Gregory. Being geometrically inclined, she observes that the windows form a 16×16 grid. Excited to beautify the dorm even further, she decides to decorate each of the 256 windows with empty flower boxes. Each flower box can only be put by one window and all the flower boxes will stay in their respective window.

(a) Now, Grace is ready to plant flowers in the boxes. She decides that she wants to distribute 16 indistinguishable roses to her boxes such that no two rose-filled flower boxes are in the same row or column. How many ways can Grace fill the flower boxes with roses?

(b) Grace is proud of her decor so far and shows off her work to Daniel. However, Daniel suggests that Grace use 16 different types of distinguishable flowers instead. After removing

all the roses she just planted, how many ways can Grace replant the distinguishable flowers such that no two flowers are in the same row or column?

5. [12 pts] Senior Power

To celebrate the start of her last year at Penn, Cindy writes all the integers from 1 to 2025 (inclusive) *in order* in one big line on one of the blackboards in Towne. Unbeknownst to her, Thomas has the same idea, and writes each of the integers from 1 to 2025 *in an arbitrary order* in a line directly under Cindy's numbers. Ria (who happens to be walking through the classrooms of Towne one day) sees the two lines of numbers, and decides to multiply the differences between each pair of numbers on the top and bottom. She wants you to guess the parity of the output. Luckily for you, you're a CIS 1600 student, and can prove with certainty that the product will always be even!

Let $x_1, x_2, \ldots, x_{2025}$ be a permutation of the numbers from 1 to 2025. Prove that the product

$$(x_1 - 1)(x_2 - 2) \cdots (x_{2024} - 2024)(x_{2025} - 2025)$$

must be an even number.

6. [8 pts] Mohit's Shah-pes

Mohit is still getting used to Penn's campus, and he's always late to class because he always gets lost. Being an expert problem-solver, he decides to place strings across campus that he can follow to help him find his way around. He places large poles at the edges of campus such that each pole forms the corners of a convex polygon with n sides. Recall that a convex shape is one where all diagonals lie entirely within the polygon. Next, Mohit hangs bright neon strings between every non-adjacent pair of poles. Assuming that no more than two strings intersect at the same point, how many intersections do the strings have at interior points?

7. [10 pts] Housing Havoc

It's time to select next year's housing! After frantically refreshing MyHomeAtPenn, you log onto the website 1 minute too late to find that the three high rises are already almost full. Harrison has enough room for 6 more people, Rodin has enough room for 5, and Harnwell has enough for 4. Unfortunately, there are $n \geq 30$ people remaining (yourself included) trying to get one of the slots. Suddenly, the Combinatorics Genie magically appears in front of you, offering to guarantee you a spot in one of the high rises if you can calculate how many ways the remaining slots can be allocated. Time is ticking, so hurry and find the number of ways in which the 15 openings across the 3 high rises (6 for Harrison, 5 for Rodin, and 4 for Harnwell) can be filled up from the pool of n prospective tenants!