CIS 1600 Assigned: August 29, 2024

Homework 1H

Due: 11:59PM EDT, September 3, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, September 3, 2024. To receive full credit all your answers should be carefully justified. **Additionally, make sure to fill out the Gradescope Policy Quiz!**

Please make note of the following:

A. LaTeX: Normally, we require all solutions to be typeset in LaTeX. We have provided a LaTeX primer video on Piazza and on the course website under the 'resources' tab, and have provided a template. From this homework on, we will only be accepting solutions that use the provided template.

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: You may not collaborate with anyone via any means.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [12 pts] No Story No Name :(

Give answers to the following questions. **You do not have to show your work for this**

question.

Note: If enumerating a set to answer set-related questions, please make sure to put the items in your set in curly braces {}.

(a) Write the following sets explicitly, i.e. list the members of these sets.

i. $\{x \mid x \text{ is a square of an integer, } x \text{ is odd, and } x < 190\}$

ii. $\{x \mid x \text{ is an integer such that } x^2 = 34\}$

iii. $\{x \mid x \text{ is a real number such that } x^2 - 40 = 9\}$

iv. $\{x \mid 3x \text{ is a positive integer less than 45 and } 7|x\}$

(b) Use the set builder notation to give a nontrivial description of each of these sets (nontrivial means that your solution should not simply enumerate every element).

i. $\{8, 11, 16, 22, 24, 32, 33, 40, 44\}$

ii. $\{4, 19, 44, 79, 124\}$

iii. $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

(c) What is the cardinality of each of the following sets?

i. $\{a, \{\{a\}\}\}$

ii. $\{\{p, m\}, \emptyset\}$

iii. $\{d, \{d\}, \{d, \{d\}\}\}$

(d) Determine whether each of the following is true or false.

i. $\emptyset \subseteq \{x\}$

ii. $\emptyset \in \{x\}$

iii. $x \subseteq \{x\}$

iv. $\{x\} \in \{x\}$

v. $\{x\} \subseteq \{x\}$

vi. $\{x\} \in \{\{x\}\}$

(e) What is the power set of $\{x, y, z\}$, where x, y, and z are distinct elements? What is the cardinality of the powerset?

(f) Find two sets A and B such that $A \in B$ and $A \subseteq B$.

2. [8 pts] Relay Rivalries

Tired of being CIS 1600 TAs, Kevin, Daniel, Sophia, Suzzy, and Olivia train for the new 5x1600m race in the 2028 Summer Olympics. However, they each want to be the first person to run, so they establish a procedure to determine the order of the competitors. Their coach, Dilini, has them line up in alphabetical order and tells them the following: the first two people must flip a coin to decide who gets to be the first person in the relay. The winner of the coin flip is assigned to the first spot in the relay, while the loser must stay to play the next person in line. The person who wins the next flip gets to run second, while the loser must again stay to play the next person in line. This process continues until there is only 1 TA left, who is the anchor. In how many orders can the 5 TAs run the relay?

3. [10 pts] A Big Task for a Little Woman

President Dilini, leader of the great country of CIS-Land takes great pride in her country's entry into the 2026 Winter Olympics and wants to ensure all n of her country's citizens will be able to compete in one of the 116 medal events, where n is a positive integer. To decide which events to train her n athletes for, she makes them line up next to each other in a row. She creates n uniforms, each being one of 116 available colors, for her athletes to wear. She decides that she'd like the colors of the uniforms to be symmetric about the middle of the n athletes. For example, if $n = 4$, then Dilini could color her athletes Blue-Orange-Orange-Blue; if $n = 7$, she could color them Green-Purple-Red-Green-Red-Purple-Green. She's flexible with which events her athletes compete in, so she doesn't require all 116 colors be used. Unfortunately, Dilini must rewatch her favorite movie, Little Women, for the 1600th time, so she needs your help to figure out the math. How many different ways are there to color the row of n athletes with 116 colors such that the colors are symmetric about the middle?

4. [12 pts] 1 Meter Sprint

Darren wants to find the perfect length for a race and has narrowed it down to anywhere from 1 to 1600 meters. Indecisive, he writes down all positive integers from 1 to 1600, inclusive. Next, he picks two of the values, erases them, and writes down their positive difference. He repeats this process until only one integer remains. Deep down, Darren knows that a race of length 1 meter is simply infeasible and would be dismayed if the final value were 1. Prove that the final value remaining will never be 1.

5. [8 pts] The Ultimate O-Luke-ian Champion

Famed Olympian Luke is basking in the glory of his many Olympic wins. To properly display his accomplishments, he decides to arrange his medals in a circle on the wall. Being the superior athlete that he is, he naturally has only gold or silver medals, and for each medal, he makes the neighboring medals a different color than itself. If Luke uses 53 gold medals in his circle, how many silver medals are there in his circle?

6. [10 pts] Victor's False Victories

Victor has always dreamed of winning an Olympic medal. Unfortunately, his dreams were crushed when he realised he didn't like any sports. Instead, he decides to go to his local Walmart and buy himself some medals so he can still feel like a winner. The Walmart has 11 gold medals, 6 silver medals, and 14 bronze medals. All medals of the same type are indistinguishable from each other. Victor wants to buy at least one medal, and he has enough money to buy all 31 medals. How many different ways can Victor choose to buy the medals?

7. [10 pts] Killer Hurdles

The year is 2040, and Olivia Hu-lloway is leading the US Olympic Trials for hurdling. Sinister as she is, Olivia poisons the top of one of the 500 hurdles (it's a really long race) so anyone who runs into the poisoned hurdle will suffer crippling pain **exactly one hour later**. However, an hour and two minutes before the event, she changes her mind and decides to remove the poisonous hurdle, only to realize that she has forgotten which hurdle it is! Olivia quickly recruits 9 of her subordinates who would be willing to endure crippling pain to test all 500 hurdles.

How should Olivia organize her subordinates to identify the poisonous hurdle before the trial takes place? Justify your answer.