CIS 1600 Assigned: December 2, 2024

Homework 15T

Due: 11:59PM EDT, December 4, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, December 4, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Ed.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

---

1. [8 pts] A Pear-fect Present

On the first day of Christmas, the 1600 TAs find a mysterious package dropped off in front of their grading room. Upon opening it, they realize it's a magical Partridge in a Pear Tree! The partridge begins zooming around campus with pears in its mouth, dropping a total of $n > 2$ pears off at various locations.

Interestingly enough, the partridge has also left m strings of Christmas lights between some pairs of pears such that none of them cross each other. Once the partridge has returned to its pear tree, the 1600 TAs scramble to survey the beautiful masterpiece the partridge has created. Let $d > 2$ be the length of the smallest cycle (in terms of the number of Christmas light strings). Victor notices that the total number of Christmas light strings m is greater than d. Help the TAs show that $m \leq \frac{d}{d-2}(n-2)$.

2. [10 pts] Best Gift Ever!!

On the second day of Christmas, the CIS 1600 TAs have decided to gift the students **Homework 15T Q2** along with instructions on how to earn full points.

Instructions: Provide finite sets X and Y and functions $f : X \to Y$ and $g : Y \to X$ such that *all* of the following conditions are simultaneously satisfied:

- $g \circ f$ is a bijection,
- $g \circ f$ is different from the identity function,
- f is not a surjection,
- g is not an injection, and
- X has exactly 2 elements.

Justify why your $g \circ f$ is different from the identity function, why your f is not a surjection, and why your g is not an injection.

3. [12 pts] Race Conditions

On the third day of Christmas, CIS 1600 gave to the TAs a chance to skip all their finals and end the semester with straight As. In order to win this golden ticket to happiness, the $k \geq 2$ TAs must compete in a race, and only if they win can they receive their reward.

The race works as follows: The k TAs all begin at the same starting line. The finish line can be reached in n steps. Each round, Alex H-Oh H-Oh H-Oh (the team captain for the TAs who doesn't participate in the race herself) chooses two disjoint and nonempty sets of TAs X and Y from the TAs that are still in the race. Note that X and Y do not need to contain every TA. Rajiv (who doesn't want every TA to simply cruise through school) then removes all TAs from

one of X or Y from the race. At the same time, each TA in the set that Rajiv did not choose moves up one step.

Alex H-Oh H-Oh H-Oh and the TAs win their special reward if any TA ever takes n steps and reaches the finish line. Rajiv wins if there is only one TA remaining and they have not reached the finish line, since Alex cannot split a single TA into two disjoint nonempty sets.

(a) Provide a strategy that guarantees Alex H-Oh H-Oh H-Oh and the TAs a win when $k \geq 2^n$. You do not need to prove your answer.

(b) Use the probabilistic method to show that there must exist a way for Rajiv to win when $k < 2^n$. You should not provide the strategy – you only need to show that one must exist using the probabilistic method.