CIS 1600

Homework 14T

Assigned: November 25, 2024

Due: 11:59PM EDT, December 2, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, December 2, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Ed.

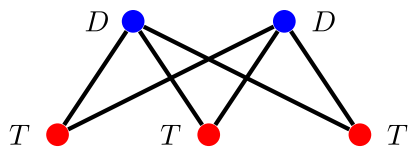
E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [18 pts] Happy Thanksgiving!

It's finally Thanksgiving, and Rajiv's Thanksgiving potluck will commence! He has invited m of his favorite TA's, and there are n classic Thanksgiving dishes on the table to be eaten. Since Thanksgiving is the time for gratitude and sharing, Rajiv wants to ensure that each TA will try each dish (and vice versa, each dish will be eaten by every single TA). Since Rajiv loves graphs, he decides to visualize the process. Below is an illustration with $n = 2$ and $m = 3$, with each edge representing a TA eating a dish.



(a) Worried that the dishes will run out and the TA's will eat too much, Rajiv decides to restrict the potluck system. He wants to restrict TA's from eating certain dishes such that the resulting potluck graph no longer has any cycles but still connects all $n + m$ vertices, as he still wants everyone to eat their share. Let D be the number of edges that Rajiv removes. Prove that D is divisible by $m - 1$. Assume $m, n > 1$.

(b) Assume that $m > n$. Prove that the length of a longest path in the original potluck graph is even.

2. [12 pts] 1600 Misses Their Flight

One quiet Thanksgiving Eve, Jonathan-ksgiving is watching the news. Suddenly the reporter, Er-traf-ic Yu, brings breaking news about air traffic. Each of the n airports across the world has at least $0 < 2k < n$ undirected routes between other airports (with each pair of airports having at most 1 route between them). Yet due to recent budgetary struggles, seemingly coming from political instability after the fall of Big Gandhi, each airport can only service at most one route on Thanksgiving Eve.

Jonathan-ksgiving knows that a lot of routes will be closed off, but he's curious about how many routes will remain open. Prove that k routes can still be open such that no airport needs to handle more than 1 route.