CIS 1600 Assigned: November 21, 2024

Homework 13H

Due: 11:59PM EDT, November 25, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, November 25, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Ed.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. **[14 pts] Thanksgiving Turkeys**

   As part of CIS 1600's Thanksgiving celebration, Grace S. and Grace Y. both plan on roasting turkeys. However, the ovens they will use are very peculiar: Every minute on the dot (i.e. at exactly 1, 2, 3, etc. minutes), Grace S's oven has a probability p of being done with cooking the turkey, while Grace Y's oven has probability q of doing the same. Let X and Y be random variables denoting the number of minutes it takes for Grace S and Grace Y's turkeys to be done cooking, respectively.

   On the day of the celebration, the two Graces discover that Rajiv will arrive in $k \in \mathbb{Z}^+$ minutes, so they immediately begin roasting their turkeys. They realize that it would be ideal if the turkeys were as fresh out of the oven as possible when Rajiv arrives, so they wish to know the probability that the first oven that finishes will finish the exact minute Rajiv arrives. In other words, find $\Pr[\min(X, Y) = k]$.

   Note that for this question, you do **NOT** need to define a sample space.

2. **[14 pts] Thanksgiving Potluck!**

   It's Thanksgiving, and Dilini has decided to host a CIS 1600 Thanksgiving potluck! For each of the n food items that the TAs have brought, Dilini asks the owner of that food item which other food items they would be willing to place their food with. After analyzing the responses, Dilini models the responses as a relation R, where $(f_1, f_2) \in R$ if the owner of f_1 is willing to put their food with f_2. If $(f, f) \in R$, that means the owner of f is willing to put their food item f by itself.

   Surprisingly, she realizes R is an equivalence relation with $n - 2$ equivalence classes; moreover, no equivalence class contains exactly 3 elements. Given all this information, how many elements are there in the set R?

3. **[16 pts] Too Many Cooks in the Kitchen**

   Vincent is hosting Thanksgiving dinner for the entire CIS 1600 staff! Everyone is excited to contribute their culinary skills to create a mouthwatering meal. On Thanksgiving Eve, Vincent starts planning who should cook which dish to ensure the day runs smoothly. He represents the set of distinct dishes as D and the set of distinct TAs as T. He knows that each TA knows how to make some (possibly distinct) positive number of dishes in D. He also notes that for any given TA, the number of dishes that they know how to make is greater than or equal to the number of TAs that any one dish could be made by. With this in mind, Vincent wants to make $|T|$ assignments of dishes to TAs such that every TA makes one dish, no dish is made by more than one TA, and every TA knows how to make the dish they are assigned. Prove such an assignment exists to help Vincent make this the best Thanksgiving ever!

4. [12 pts] Pie Day on Thanksgiving?

To ensure that there will be plenty of dessert to enjoy at the CIS 1600 Thanksgiving potluck, Dilini decides to also host a pie competition and invites the TAs to bake their best pies! After all the pies are ready, Dilini arranges them on tables for display. Each pie is placed on exactly one table. To organize the display, Dilini defines A to be the set of pies in the competition and considers R, the relation "on the same table as" on A (i.e. $(x, y) \in R$ iff x is on the same table as y). Additionally, the pies are arranged with the following condition:

> If pie x is placed on the same table as pie y and pie y is placed on the same table as pie x, then x and y must be the exact same pie.

To make the competition even more exciting, Dilini announces that whoever correctly calculates $|R|$ first will get to have the entire winning pie! Being the ultimate pie enthusiast, Jonathan Pie is determined to win this prize. Help Jonathan by finding $|R|$. Prove your answer.

5. [14 pts] Thanksgiving Table Tactics

David Fu is organizing his extended family's Thanksgiving dinner! As an expert planner, one of his responsibilities is deciding seating arrangements for the dinner. There are only two tables, so he must assign each of the n family members (including David) to one of the two tables. However, his family isn't without tension. There are m distinct (not necessarily disjoint) pairs of people in the extended family who, if seated at the same table, would argue unendingly the whole dinner, so David wants to minimize putting these people together. He would like to ensure that for at least $\frac{m}{2}$ of the pairs of family members who will argue, the two members of the pair are seated at different tables (even if this means the tables are of uneven size).

Help David's Thanksgiving planning and prove there exists a configuration of two tables such that this condition is satisfied, to minimize Thanksgiving familial strife.