

Homework 12T

Due: 11:59PM EDT, November 13, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, November 13, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

- A. \LaTeX :** Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.
- B. Standard Deductions:**
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- C. Solutions:** Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.
- D. Collaboration:** Please make sure to strictly follow our collaboration policy as clarified on Ed.
- E. Citations:** All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.
- F. Outside Resources:** Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.
- G. Late Policy:** We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.
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1. [8 pts] The Great Pyramid of Eternal Harmony

After an awe-striking trip to the Great Pyramid of Giza, David Khu-Fu plans to build an even more magnificent Pyramid himself. He decides to be extravagant and constructs x distinct King Chambers and y distinct Queen Chambers in his Pyramid.

To make his Pyramid more impressive, David Khu-Fu calls upon O-Sid-ris to enchant the Pyramid with a special spell, giving each chamber a distinct aura. Being the goddess of harmony, Ma'at-llika takes notice of David's dedication to the Pyramid and decides to add her own touch to the enchantment: she weaves an extra layer of magic into the spell such that the aura of each King Chamber harmonizes with exactly k distinct Queen Chambers, and the aura of each Queen Chamber harmonizes with exactly k distinct King Chambers. However, no King Chambers harmonize with other King Chambers and no Queen Chambers harmonize with other Queen Chambers.

Prove or disprove that this magical harmonization implies that $x = y$. You may assume that $0 < k \leq x, y$.

2. [14 pts] Stochastic Sid

Sid is planning to return to his homeland and visit his birthplace, Agra. He is excited to see the Taj Mahal, but he wants to seem local so they don't charge him the foreigner fee. To do so, he has to write an essay in Hindi about why he loves India and the Taj Mahal. Given that he doesn't know how to write Hindi, Sid decides to pick English uppercase letters independently and uniformly at random with replacement. He will continue picking letters until he picks a T directly followed by a M (for Taj Mahal). Sid is unsure whether this procedure is likely to result in too few or too many letters in his essay. What is the expected length of Sid's essay (in English letters)?

Hint: Try applying the memoryless property.

3. [8 pts] William of Guo-l's Last Theorem

Legend has it that William of Guo-l retreated to the British Isles and made his final stand at Stonehenge. However, in the hopeless battle, Will realized that he may be able to harness the mathematical power of Stonehenge to halt the Roman advance. With time running out, Will was forced to solve the following questions about relations to curse the Roman legions and send them back to the land of the Franks. With the Guo-lish lines completely broken, Will made a desperate final act and carved the proofs as quickly as he could into a stone tablet. In an instant, fear was struck into the Roman legions and they fell into full retreat.

Help recreate the famous proofs William of Guo-l solved at the battle of Stonehenge.

Let A be a set and R a relation on A that is both symmetric and antisymmetric.

- (a) Prove that if $a, b \in A$ and $a \neq b$, then $(a, b) \notin R$.
- (b) Is R always reflexive? Prove your answer or give a counterexample.

Now, let S and T be distinct relations on the set A that are both transitive.

- (c) Is $S \cup T$ always transitive? Prove your answer or give a counterexample.
- (d) Is $S \cap T$ always transitive? Prove your answer or give a counterexample.