CIS 1600 Assigned: November 4, 2024

Homework 11T

Due: 11:59PM EDT, November 6, 2024

This homework is due electronically on Gradescope at 11:59PM EDT, November 6, 2024. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. LaTeX: Please typeset all your answers in LaTeX based on the template we provide for you. Failure to do so will result in a 0 for the homework.

B. Standard Deductions:

- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.

C. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 1600.

D. Collaboration: Please make sure to strictly follow our collaboration policy as clarified on Piazza.

E. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

F. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

G. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

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1. [8 pts] Hat Attack

On his way to class one day, Jonathan is ambushed by a group of k garden gnomes, where $k \geq 3$. These gnomes form a circle around Jonathan facing inwards and toss their hats once. Each gnome, independent of the others, tosses their hat such that there is a 1/3 probability the hat lands on the head of the gnome to their immediate left, and a 2/3 probability that the hat lands on the head of the gnome to their immediate right. In order to break out of the ambush, Jonathan needs to correctly calculate the expected number of hat-less gnomes at the end of the hat tossing. Help him out!

Note that multiple hats can land on the same gnome's hat, without affecting anything.

2. [12 pts] Avalanche Adventures

Winston is mountain-climbing in the Himalayas and approaches a local village warning him of signs of yetis. Hearing this, he decides to shorten his planned itinerary out of the n possible mountains he can climb, where $n \geq 3$. He decides to visit exactly k of these mountains, where $3 \leq k \leq n$. Winston has access to a helicopter that can fly him to any peak to begin his trek. However, for the rest of the trek, he will walk from peak to peak using the mountain passes. Every pair of peaks is connected by a pass, but each pass has an independent $\frac{1}{2}$ chance of being closed due to avalanche predictions, meaning Winston cannot use it. Throughout his trek, he must visit k distinct peaks, end at the same peak he began on, and never visit the same peak twice (except for the starting peak).

Let T be the number of possible treks he could go on, with two treks considered equal if they use the same passes, even if Winston begins at a different peak or if he traverses the passes in the opposite direction. For example, $a \to b \to c \to a$ is equal to $b \to c \to a \to b$ (different starting peak), which is equal to $b \to a \to c \to b$ (reversed). However, $a \to b \to c \to d \to a$ is not the same trek as $a \to c \to d \to b \to a$. Find the expected value of T.

3. [10 pts] Thomas and the Mino-Toro

Unfortunately, it's that time of year again. Jan Mino-Toro has demanded a tribute of 1600 staff to the island of Dis-Crete. Thomas has volunteered to slay the beast and take glory and honor back to Athens. But first, he has to overcome the Labyrinth.

There are $n \geq 1$ rooms, where undirected passageways between rooms in the Labyrinth such that at most one passageway connects any two rooms. It may or may not contain cycles of rooms joined via passageways, and there may or may not exist some pairs of rooms not reachable from the other by a sequence of passageways. Luckily, Thomas knows a secret about the Labyrinth: there is no set of three rooms where a passageway exists between every pair of rooms in the set.

Prove that the number of passageways is at most $\frac{n^2}{4}$ to help Thomas defeat Jan Mino-Toro.