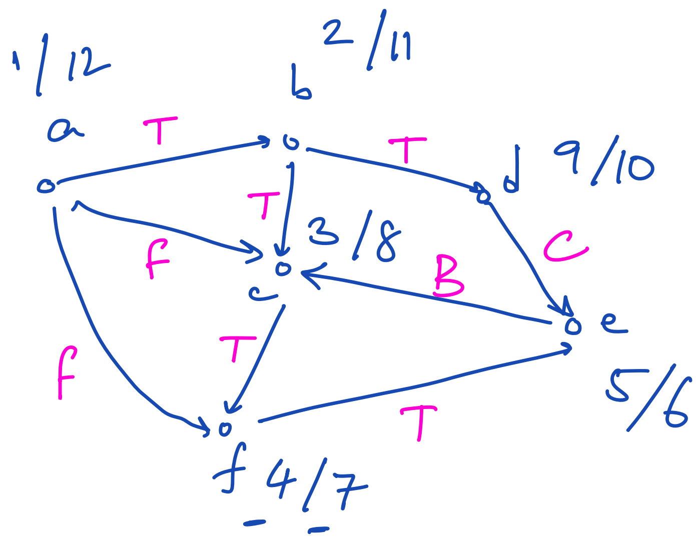
- NO OH TODAY

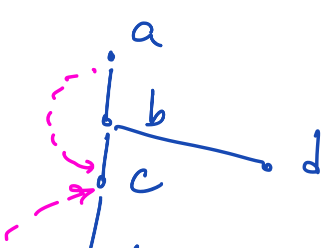
- In-person lecture on Thu.

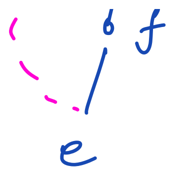
- Exam 1

    - Q.1, Q.2, Q.3

Depth first search.







<u>Theorem</u>: DFS on an undirected graph yields only tree edges & back edges.

<u>Proof Sketch</u>: $e = (u,v)$ be an arb. but particular edge in G.

WLOG, let $d[u] < d[v]$.

<u>Claim</u>: v is a descendant of u in the DFS forest.

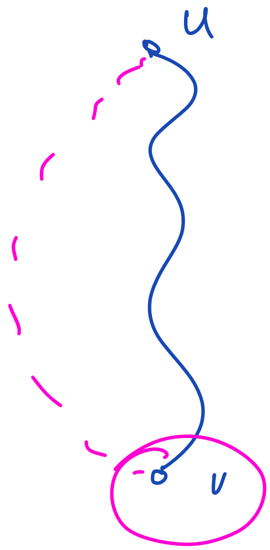
Why? At $d[u]$ there is a WP from u to v in G.

WPT $\Rightarrow$ the claim

<u>Case I</u> : v is a child of u in the DFS forest

- e is a tree edge.

<u>Case II</u> : v is a descendant of u in the DFS forest but not a child of u.



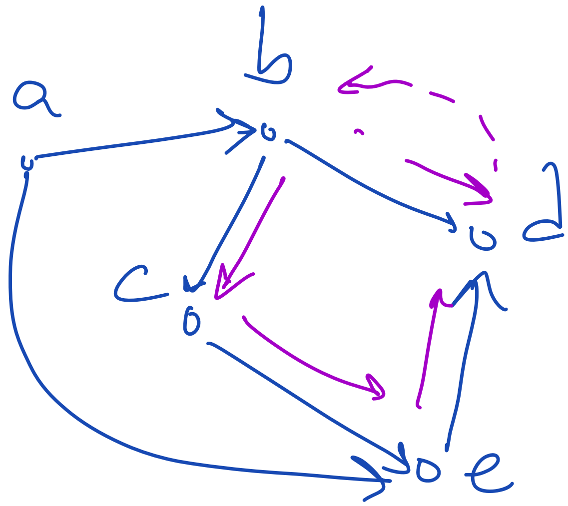
e is a back edge. ✓

<u>Topological Sort</u> -

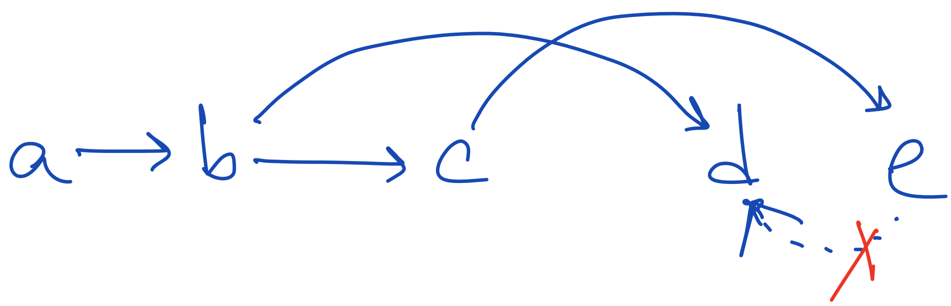
<u>Input</u>: Directed Acyclic Graph $G = (V, E)$

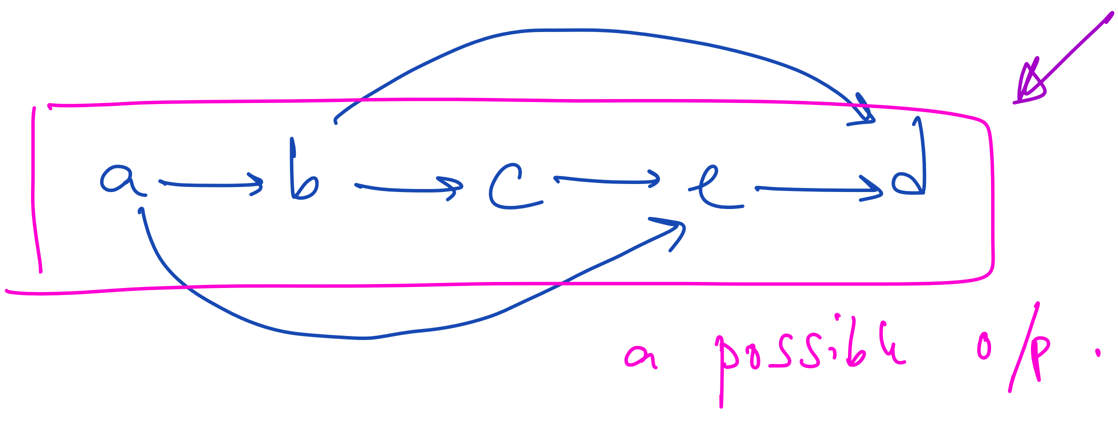
Obj: To obtain a <u>topological sort</u> of G.

ordering of the vertices in a row s.t. all edges of G go from left to right.



no directed cycle.

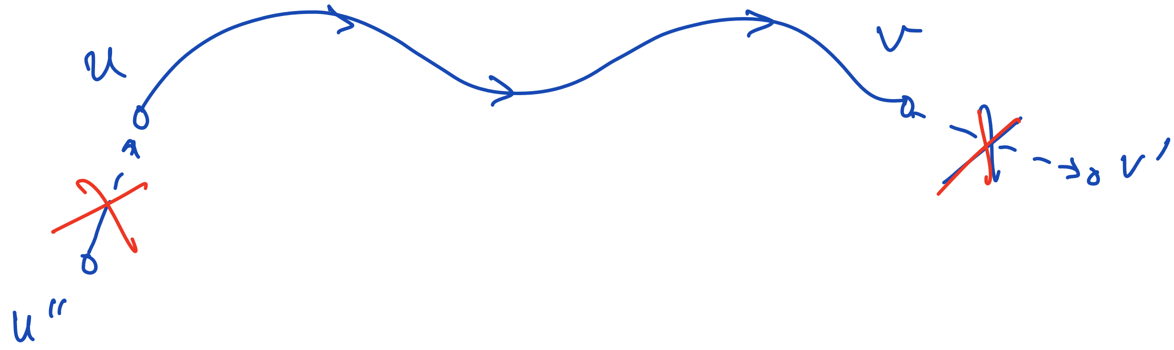




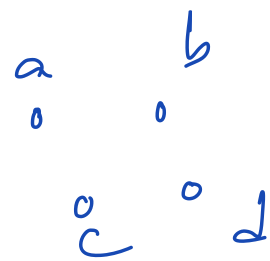
a possible o/p.

Source : vertex in G with indeg $= 0$.

Sink : " " " with outdeg $= 0$.



TS(G)



$u \leftarrow$ any source in G $\leftarrow$ $n + n-1 + \cdots + 1 = \Theta(n^2)$

$G' \leftarrow G - u$ $\quad O(n+m)$

$L \leftarrow TS(G')$ $\quad \rightarrow T(n-1)$

o/p u followed by vertices in L.

Running time: $O(n^2)$ $\leftarrow$

Runtime recurrence :

$$T(n) = T(n-1) + O(n)$$

Clever implementation

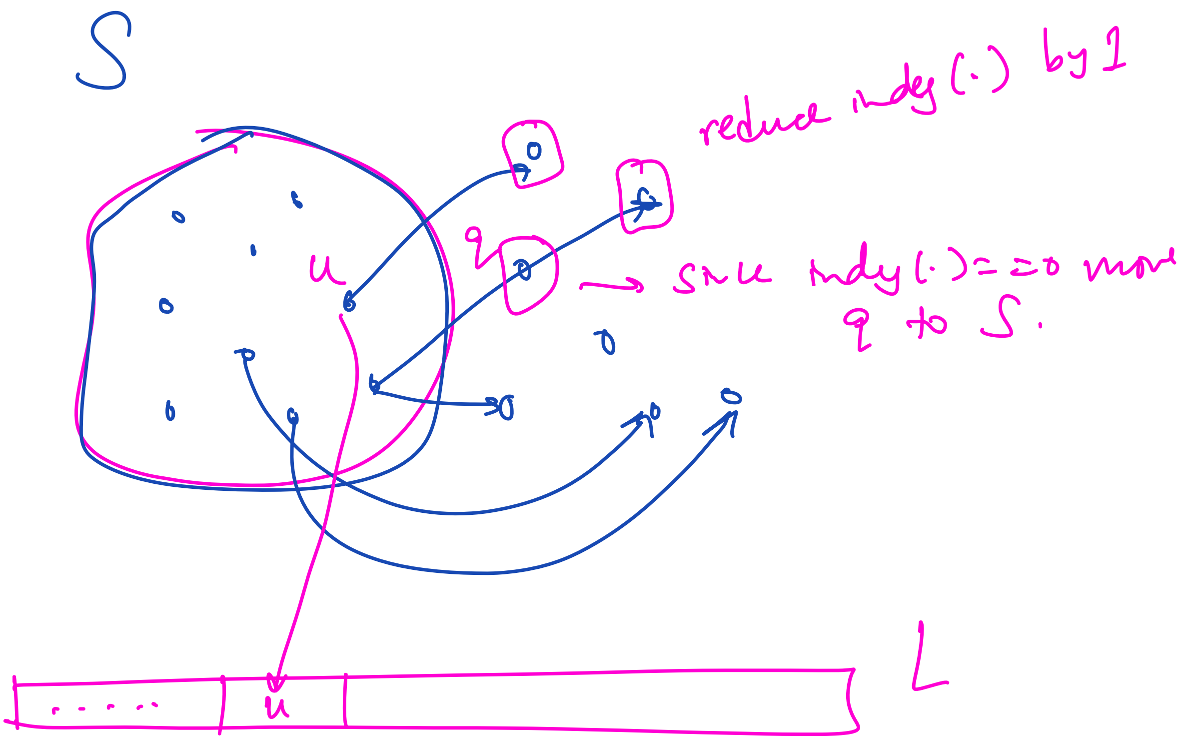
1. $S \leftarrow$ all sources in G. $\rightarrow O(n+m)$
2. **while** $S \neq \emptyset$ **do**
3. $\quad u \leftarrow$ any vertex in S ; $S \leftarrow S \setminus \{u\}$
4. $\quad$ append u to L
5. $\quad$ **for** each $v \in N(u)$ **do**
6. $\quad\quad$ indeg(v)--

$\sum_u \text{out}(u) = O(m)$

7. if $\text{indeg}(v) = 0$ then

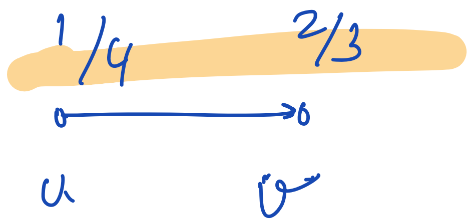
8. $S \leftarrow S \cup \{v\}$

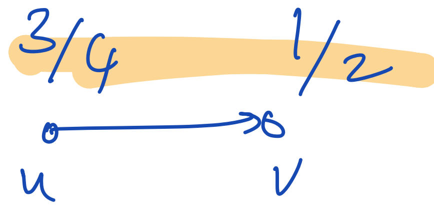
9. o/p L.



Running time: $O(n+m)$.

Alt alg.





1. DF(G) $\longrightarrow O(n+m)$

2. order vertices in $\downarrow$ order of $f[\cdot]$.

$O(n \lg n)$

$O(n \lg n + m)$.

Modifying DFS & moving a vertex at the start of the o/p (prepend) when the vertex finishes in DFS can yield $O(n+m)$ time.

Correctness We want to show that

the alg. works.

Proof: Let $e = (u, v)$ be any edge in G.

We want to show that u appears to the left of v in the o/p. That is,

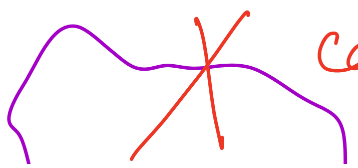
T.S.T. $f[u] > f[v]$.

Case I: $d[u] < d[v]$

By the WPT, v is a desc. of u in the DFS forest.

By the PT, $d_u < d_v < \boxed{f_v < f_u}$

Case II: $d[v] < d[u]$.

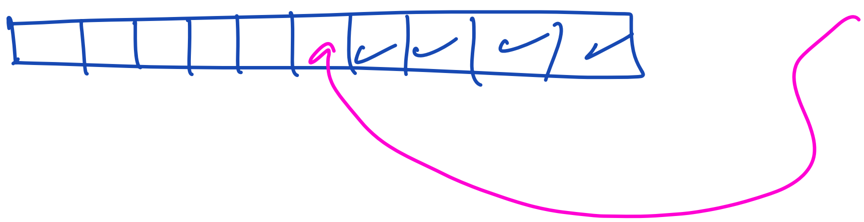


Cannot exist.

~~By the WPT, at time $d[v]$, there is a WP from v to u in G $\Rightarrow$ ..... $f[u] > f[v]$.~~

At $d[v]$ there is no WP from v to u in G & hence u is NOT a desc. of v in the DFS forest. By the PT,

$\Rightarrow$ $f_u > f_v$. ✓.



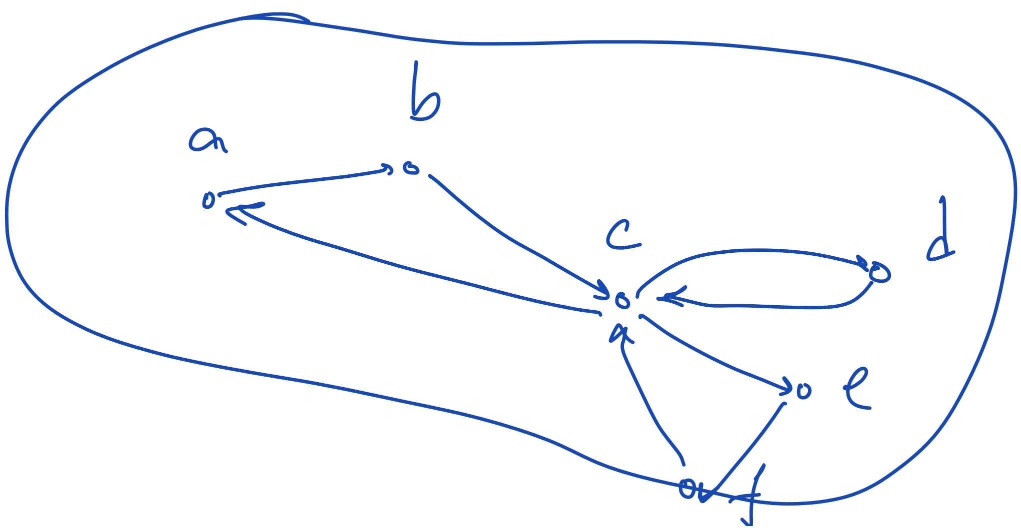
Strongly Connected Components (SCC)

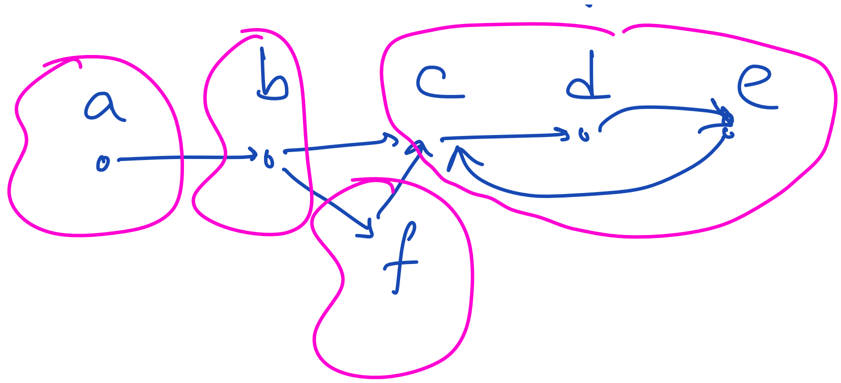
Input: Directed graph $G = (V, E)$.

O/p: All SCCs of G.

$H = (V_H, E_H)$ is a SCC of $G = (V, E)$ if

- H is a subgraph of G
- $\forall\, u, v \in V_H,\ u \neq v,\ u \rightsquigarrow v$ & $v \rightsquigarrow u$ in G
- H is maximal.





A DAG on n vertices has exactly

$\underline{\quad n \quad}$ SCCs.