CIS 1210 — Data Structures and Algorithms
Homework Assignment 6

Assigned: March 4, 2025 **Due:** March 24, 2025

Note: The homework is due **electronically on Gradescope** on March 24, 2025 by 11:59 pm ET. For late submissions, please refer to the Late Submission Policy on the course webpage. You may submit this assignment up to 2 days late.

A. **Gradescope**: You must select the appropriate pages on Gradescope. Gradescope makes this easy for you: before you submit, it asks you to associate pages with the homework questions. Forgetting to do so will incur a 5% penalty, which cannot be argued against after the fact.

B. **LaTeX**: You must use the LaTeX template provided on the course website, or a 5% penalty will be incurred. Handwritten solutions or solutions not typeset in LaTeX will not be accepted.

C. **Solutions**: Please write concise and clear solutions; you will get only a partial credit for correct solutions that are either unnecessarily long or not clear. Please refer to the Written Homework Guidelines for all the requirements. Ed will also contain a complete sample solution in a pinned post.

D. **Algorithms**: Whenever you present an algorithm, your answer must include 3 separate sections. Please see Ed for an example complete solution.

1. A precise description of your algorithm in English. No pseudocode, no code.
2. Proof of correctness of your algorithm
3. Analysis of the running time complexity of your algorithm

E. **Collaboration**: You are allowed to discuss **ideas** for solving homework problems in groups of up to 3 people but *you must write your solutions independently.* Also, you must write on your homework the names of the people with whom you discussed. For more on the collaboration policy, please see the course webpage.

F. **Outside Resources**: Finally, you are not allowed to use *any* material outside of the class notes and the textbook. Any violation of this policy may seriously affect your grade in the class. If you're unsure if something violates our policy, please ask.

1. [20 pts] The Mystery at MacDougall Manor

In the shadowy halls of the 1210 Staff's grand estate, the MacDougall Manor, a chilling plot is unfolding. Strange happenings have unsettled its resident 1210 TAs — creaks in the floors, creepy whispers in the walls, and the growing sense that something supernatural is unraveling the estate from within.

The manor is composed of $n \geq 2$ rooms which are connected by m undirected hallways. Each hallway connects exactly two rooms, and the hallways are organized in such a way that any pair of rooms are reachable from one another.

The TAs realize that in order to protect the estate, there are k haunted rooms that must be blocked off from the rest of the manor. Even the hallways incident to these rooms may be haunted, so they must be avoided as well.

Detective Joe has been called in to solve the mystery and find exactly which k rooms must be blocked off without disrupting the estate's delicate balance. His task is clear — after blocking off the k rooms and their incident hallways, Detective Joe wants every pair of the remaining rooms to still be reachable from one another.

With a ticking clock and the estate's fate hanging in the balance, Detective Joe must uncover the truth soon. Design an $O(n + m)$ algorithm for Detective Joe to find the k such rooms that he should block off.

Note that k is a positive integer $< n$ that cannot be treated as a constant.

No proof of correctness is necessary, but please do justify your runtime.

2. [20 pts] Detective Phillip Holmes

Tragedy has struck! At the annual CIS 1210 retreat, someone has stolen the coveted Lost A.R.K. (Algorithmic Rationalization Key), a powerful artifact that enables unparalleled problem-solving skills and computational efficiency – and it's up to detective Phillip Holmes to figure out which TA is the culprit. To find the traitorous TA, he must analyze the retreat house to look for clues. The house contains n rooms, which are connected by doorways. The doorways are two-way, meaning that he and the TAs can enter in either direction through the doorway. Note that a single doorway connects exactly 2 rooms, and that it is possible to walk from any room x to any other room y via a sequence of doorways. Furthermore, there is only one sequence of doorways between any two rooms.

Phillip notices that each of the rooms contains one clue about who stole the Lost A.R.K., and that 2 of the rooms contain a super-clue, which must be combined to form a secret message. Unfortunately, the two super-clues are located in the two rooms that are farthest away from each other (i.e. separated by the greatest number of rooms). Design an $O(n)$ algorithm that returns the pair of rooms with the longest path between them.

No proof of correctness is necessary, but please do justify your runtime.

3. [15 pts] Gabe's Spring Break Galactic Quest

Gabe, the head TA, has entrusted Vedha, Tanvi, and Katherine with a mission of cosmic importance during spring break. Their destiny lies on a distant planet, exactly n miles away, where a legendary treasure is hidden. This treasure, once recovered, is believed to hold the power to save their beloved planet "$\alpha - 121$" from imminent destruction at the hands of the notorious TA, Gaurav, who is from a rival planet.

Equipped with a state-of-the-art starship, the trio embarks on their quest. The ship is powered by an extraordinary propulsion system featuring p distinct propellers. Each propeller, when activated, propels the vessel forward by a unique positive integer distance, p_i light years (no two propellers move the ship by the same amount). Although each propeller can be used repeatedly, only one can be activated at a time, and every activation adds a cost of 1 *unit* to the total propeller usage. For example, activating propellers p_a, p_b, p_a in that order results in a total usage cost of 3.

Given n, p, and the distance that each propeller can travel in light years, help Katherine, Vedha, and Tanvi by designing an algorithm that will help them find the exact sequence of propeller activations that minimizes the propeller usage, while ensuring the starship travels exactly n miles—with no overshooting. If it is impossible to reach the target distance using the available propellers, the algorithm should output "*NOOOO*".

Your algorithm should run in $O(pn)$ time, and should only use modifications to graph algorithms.

4. [20 pts] Darsh's 3200 Notes

Oh no! Tanay forgot to take lecture notes in 3200 and is stressed about the midterm exam. As a good Head TA, Darsh decides to help him out. However, to make sure Tanay never does this again, he spreads out all his notes, page-by-page and places them at different GSRs in Amy Gutmann Hall. He also gives Tanay a map of the AGH hallways.

The map consists of C GSRs connected by H bidirectional hallways, where each hallway connects exactly two GSRs. To collect all the notes one-by-one, Tanay needs to find a way to assign a single direction to every hallway in H. In order to save time to study for the exam, Tanay wants to make sure that he assigns directions to hallways in such a way that he creates no cycles so he never visits the same GSR again. After assigning directions to a subset $H_1 \subseteq H$ of the GSRs, he realizes he is out of time and cannot direct the remaining hallways. Let these remaining hallways be denoted by the subset $H_2 = H \setminus H_1$. Help Tanay out by coming up with an $O(|C| + |H|)$ time algorithm to assign directions to the remaining hallways in H_2 so that he does not create any cycles that allow him to visit any GSR twice. You may assume access to $G(C, H_1)$ and $G(C, H_2)$ as separate adjacency lists.

5. [25 pts] Kevin's Cupcake Culprits

For Kevin's birthday, the CIS 1210 Staff decided to have a big surprise bash! To prepare, Aaron, Advit, and Arriella were tasked with baking an assortment of cupcakes. Masters of their craft, all three of them keep a top secret baking log with the relative ordering of when each cupcake was baked and then frosted. However, these pastry pros tend to sugarcoat both their cupcakes and the truth. It is your job to investigate these three bakers and see if they are stretching the truth.

Throughout their time baking, exactly n cupcakes were baked and then frosted. You would like to hear everything that this trio has to say about these n cupcakes to see whether or not their claims make sense together.

Since you cannot access their top secret baking books, you choose to interrogate Aaron, Advit, and Arriella. For each cupcake pair that you ask about, exactly one of the bakers responds. You soon realize that the trio each have very different ways of conveying information. Specifically, for each pair of cupcakes (a, b) that you ask about, **exactly one** of the following would occur:

- Aaron would claim that, at some point, he saw a and b simultaneously being baked.
- Advit would claim that he saw a get frosted before b was baked, or vice versa.

- Arriella would stay quiet, hoping to not reveal any mischievous activities of her partners in crime. In this case, you would learn nothing about this cupcake pair.

Additionally, you know that these three are the best bakers in the world and always bake a cupcake before frosting it.

Given these $\binom{n}{2}$ responses, you must check if there are any contradictions. That is, you want to ensure that there exists some relative timeline such that all of their claims hold.

For example, the following set of claims would be contradictory:

- Claim 1: a is frosted before b is baked.
- Claim 2: c is frosted before d is baked.
- Claim 3: b and c were simultaneously baked at some point
- Claim 4: a and d were simultaneously baked at some point

Notice that you cannot assign relative times to cupcakes a, b, c and d such that all four claims hold. Give an $O(n^2)$ time algorithm that returns false if the statements are contradictory and true otherwise.

Hint: Consider constructing a graph with $2n$ *vertices.*