

**CIS 6100, Spring 2023**  
**Advanced Geometric Methods in Computer Science**  
**Project Topics**

1. Propose your own topic!
2. The proof of the Cartan von-Neumann theorem; Theorem 4.8 of diffgeom-I. Extension to the version for closed subgroups of  $\mathbf{GL}(n, \mathbb{C})$ .
3. The equivalence of the three definitions of a tangent vector. Sections 8.2, 8.3, 8.4,
4. The Brouwer degree and a proof of the “hairy-ball theorem.” Chapter 5 of Milnor’s *Topology from the Differentiable ViewPoint*.
5. The index of a vector field, the Euler characteristic of a manifold, and the *Poincaré-Hopf theorem*. Chapter 6 of Milnor’s *Topology from the Differentiable ViewPoint*. See also Do Carmo’s *Differential Geometry of Curves and Surfaces* and Guillemin and Pollack’s *Differential Topology*.
6. Covering maps, the fundamental group, the universal cover of a manifold. Section 11.2 of diffgeom-I.
7. Covariant derivative of a vector field along a curve on a surface. Explicit formula in terms of the Christoffel symbols. Slides CIS610-surfaces-slides.pdf, Do Carmo’s *Differential Geometry of Curves and Surfaces*.
8. Geodesics and the exponential map. Sections 16.1 and 16.2 of diffgeom-I.
9. The geodesics on the ellipsoid. Hilbert and von Vossien’s *Geometry and the Imagination*.
10. The Hopf-Rinow theorem and the cut locus. Section 16.3 of diffgeom-I.
11. The Riemann curvature tensor, sectional curvature, Sections 17.1 and 17.2 of diffgeom-I.
12. Riemannian covering maps and Riemannian submersions. Sections 18.2 and 18.3 of diffgeom-I.
13. The correspondence Lie-group Lie algebra; Theorem 19.20. Fulton and Harris’ *Representation Theory*.
14. Bi-invariant metrics on Lie groups. diffgeom-I, Section 21.2.

15. Connection and curvature of left-invariant metrics. diffgeom-I, Section 21.3.
16. Connection and curvature of bi-invariant metrics. diffgeom-I, Section 21.4.
17. Semisimple Lie algebras, Cartan's criterion for semisimplicity using the Killing form. diffgeom-I, Sections 21.5, 21.6. Fulton and Harris' *Representation Theory*.
18. Cartan connections, diffgeom-I, Section 21.7.
19. Proper and free actions. diffgeom-I, Sections 23.1, 23.2.
20. Riemannian submersions and coverings induced by group actions. diffgeom-I, Section 23.3.
21. Reductive homogeneous spaces. diffgeom-I, Section 23.4.
22. Naturally reductive homogeneous spaces. diffgeom-I, Section 23.6.
23. Symmetric Spaces. diffgeom-I, Section 23.8.