Edge Detection
and Multiple Scales
Edge detection
Smoothing an edge with a Gaussian
... and taking derivative and magnitude maximum
Ridge (or Box) Detection
Build a system that detects edges and features independent from scale (size)

Let us look at an “unconventional edge”: \[
\cos \omega_0 x \rightarrow g'_\sigma(x)
\]
Build a system that detects edges and features independent from scale (size)
Response is maximum when filter scale $\sigma$ matches incoming scale ($\omega_0$).
Look at edge detection as template matching

Which size of “edge template” matches the original edge?
Edge detection output for different $\sigma$’s:

As $\sigma$ increases the peaks at the edges weaken!!

$\omega_0 e^{-\omega_0^2 \sigma^2 / 2}$
Why we need a scale normalization of the filter!

Assume a scaled version of the original signal (twice as wide for $s=2$), then calculate the convolution with Gaussian:

$$I'(x) = I\left(\frac{x}{s}\right)$$

$$(I' \star g_{s\sigma})(x) = (I \star g_{\sigma})\left(\frac{x}{s}\right)$$

Scaling by $s$ and convolving with scaled Gaussian is the same as applying a Gaussian and then scaling by $s$!!!
But this is not true for the 1\textsuperscript{st} derivative!

\[(I' \star g'_{s\sigma})(x) \neq \frac{1}{s} (I \star g'_{\sigma})\left(\frac{x}{s}\right)\]
No matter what the scale of the feature, edge detector (1\textsuperscript{st} Gaussian derivative) response should be the same when the filter matches the feature scale.
Can we find (select) the intrinsic image scale?

Yes, by taking the maximum over scale!