1 Gabor filters

In this section, we will implement 2D detection using a Gabor filter. We will first play with two toy examples in 1D and 2D to understand the fundamentals, then we will attempt to have your computer play the game of “Where’s Waldo”.

1. Gabor filter in 1D

In this question we will consider the script `gabor1d_script.m` in the homework kit: it generates a 1D signal of increasing frequency `s` as well as a vector `s_w` containing ground-truth frequencies (see figure 1).

(a) [1 point] Complete the function `gaussian1d.m` that returns discrete normalized centered Gaussian of given standard deviation `sigma` and length.

(b) [2 points] Complete the function `gaborFilter1D.m` that returns a pair of Gabor quadrature filters at given spatial period `T_f` in pixels, Gaussian envelope `sigma` and length. The two filters correspond respectively to the real and imaginary part of the Gabor filter we defined in class.

(c) [4 points] Complete the script `gabor1d_script.m` to compute the filter response: `r1, r2` are the imaginary and real parts of the response, and `energy` is the magnitude of the response (complex norm).
(d) [2 points] Using the script, plot figures for two distinct values of \( T_f \) and check that the maximum response matches the ground truth.

2. **Gabor filter in 2D**

We now consider the same problem in 2D: we want to detect patterns with a specific spatial period and orientation in the image represented in figure 2.

![Figure 2: Image containing a range of frequencies and orientations.](image)

(a) [2 points] Complete the function `gaussian2d.m` that returns discrete normalized centered 2D Gaussian of given covariance matrix \( \Sigma \) and size.

(b) [5 points] Complete the function `gaborFilter2D.m` that returns a pair of Gabor quadrature filters to detect a given spatial period \( T_f \) in pixels and orientation \( \theta \) in degrees. The two filters correspond respectively to the real and imaginary part of the Gabor filter we defined in class.

(c) [4 points] Complete the script `gabor2d_script.m` to compute the filter response: \( r1, r2 \) are the imaginary and real parts of the response, and \( \text{energy} \) is the magnitude of the response (complex norm).

(d) [2 points] Using the script, plot figures for two distinct values of \( T_f \) and check that the maximum response matches the ground truth.

3. **Where's Waldo?**

Now we consider the classical game of finding Waldo (a character with a red-and-white striped shirt) in an image like figure 3. We are going to use the function `gaborFilter2D` to craft a filter for horizontal oscillation at an appropriate frequency.

(a) [2 points] Read through the script `waldo_script.m` and describe what it does in a couple sentences.

(b) [2 points] Look at the first plot (run the script up to line 28 included) and explain in a few words the formula for \( \text{im\_red} \), line 14.

(c) [6 points] Complete the function `determineStripePeriod.m` that interactively determines the spatial frequency of Waldo-like stripes in the image: you only need to complete the part computing \( T_y \) after the user clicks on a peak of the `fft`.
Figure 3: Where’s Waldo?

(d) [8 points] Complete end of the script `waldo_script.m` to compute the response of the Gabor filter and the detection mask; figure 4 shows a part of the image multiplied by `detection_mask`, which is a double matrix equal to 1 in the candidate areas and 0 anywhere else.

You can compute `detection_mask` by thresholding `energy` and by applying `imdilate` with a 'disk' structuring element of size 30, to make the zones slightly bigger and include a small portion of the image around the detected stripes: it makes it more convenient to see Waldo’s head! Read the documentation of `imdilate` and `strel` for more details.

Figure 4: Example output: the energy response is thresholded and dilated, and the resulting detection mask is applied to the image to show only candidate zones with stripes at the right frequency.
2 AlexNet

AlexNet\(^1\) popularized convolutional neural networks (CNNs) after winning the ImageNet ILSVRC 2012 challenge with a huge margin to the runner-up. It has been observed that the filters in the first layer of a CNN usually resemble Gabor filters. This question explores this idea.

1. **[5 points] Weight visualization**

   Download the pre-trained AlexNet weights here: [http://www.vlfeat.org/matconvnet/models/imagenet-caffe-alex.mat](http://www.vlfeat.org/matconvnet/models/imagenet-caffe-alex.mat). Load the weights of the first layer on MATLAB by doing:

   ```matlab
   model = load('imagenet-caffe-alex.mat');
   weights = model.layers{1}.weights{1};
   ```

   You should obtain a $11 \times 11 \times 3 \times 96$ tensor. This corresponds to 96 filters with dimensions $11 \times 11$ and 3 channels (B, G, R). Plot the filters arranged in a $12 \times 8$ grid. **Hint: normalize the filters to be between 0 and 1 for better visualization.**

   Select, from the 96 filters, examples of filters that can and that cannot be approximated with the real part of a 2D Gabor (as produced by `gaborFilter2D.m`). Explain your answer.

   Note that, since the filters are applied convolutionally, we don’t care much about the spatial position, i.e., an off-centered filtered with small support is equivalent to its centered version.

2. **[7 points] Approximating filters with Gabors**

   Consider the two filters from Figure 5. Approximate them by $11 \times 11$ 2D Gabor filters. Show your approximations and the parameters chosen.

   ![Figure 5: A couple of AlexNet filters.](image_url)

   In AlexNet, each of the $11 \times 11 \times 3 \times 96$ weights of the first layer have to be learned. If we were the replace the first layer of AlexNet with a bank of 96 Gabor filters, how many parameters would have to be learned?

---

3 Scale invariant detection

In this section, we will implement a scale-invariant blob detector that produces results as in figure 6: we will recursively apply a DoG filter to the initial image to build a 3D-matrix of responses, and we will then find local maxima in position and scale.

![Scale-invariant blob detection example](image)

Figure 6: Scale-invariant blob detection example.

1. **Approximating a LoG by a DoG**

   In this section we will experiment with approximating a Laplacian of Gaussian filter by a Difference of Gaussians. To build the intuition we will reason in 1D, i.e with a section of the filter.

   (a) **[7 points]** We want to compute and plot a LoG filter, as well as its approximation by DoG filters. Remember the following:

   \[
   \text{LoG}_\sigma = \frac{1}{(k-1)\sigma^2} \text{DoG}_\sigma, \quad \text{where} \quad \text{DoG}_\sigma = G_{k\sigma} - G_\sigma
   \]

   In DoG_script.m, we will try several values of \( k \) in \( k\_\text{range} \). Complete dog1d.m and DoG_script.m. Quickly comment on the plots you obtain.

   (b) **[2 points]** Explain why you could expect the DoG to get closer to the LoG as \( k \) tends to 1. (Remark: Note that in practice, when building a scale space using DoG, we prefer using \( k = \sqrt{2} \))

   (c) **[2 points]** In practice, when building the scale space, we intentionally forget the normalizing factor \( \frac{1}{(k-1)\sigma^2} \) and just use the differences of Gaussians. Explain why.

2. **Detecting sunflowers**

   The LoG filter in 2D is a rotationally symmetric version of the 1D filter we worked with in the previous question (the famous “mexican hat”). Because of its shape, it makes a good blob detector (a blob is a dark patch on light background). We don’t know the size of the blobs to detect a priori, so we build a scale space by applying the filter with wider and wider \( \sigma \). In practice we will approximate the LoG by a DoG. We will implement a simple (but inefficient) version where we do not downsample the image when blurring it.

   (a) **Preliminaries**

      In this section the only file you need to complete and run is blob_script.m. Make sure you:
• change the two paths at the beginning of the file: one is for the image, the other for a function
minmaxfilt used to find local maxima/minima in an \( n \)-dimensional array.

• run minmaxfilter_install.m in MinMaxFilterFolder, to compile the mex files for your plat-
form. (The homework kit already includes compiled mex files for 64-bit Linux, so you can skip this
step if you’re using that platform)

(b) [7 points] Complete the part of blob_script.m that builds the 3D matrix scales. Each slice scales(:,:,i)
contains the original image, blurred by a simple Gaussian filter of parameter \( \text{sigma}_i = \text{sigma} \times k^{(i-1)} \).

Important: we exploit the fact that the 2D Gaussian filter is separable. Instead of convolving the image
with a 2D Gaussian, build a 1D Gaussian filter \( \text{g}_i \) (using gaussian1d.m), convolve the image with \( \text{g}_i \)
and then with \( \text{g}_i' \) (transpose of \( \text{g}_i \)): this has the same effect as convolving with the 2D filter but is more
efficient.

(c) [7 points] Complete the part of blob_script.m that filters the local maxima in the scale space according
to their response. Here we want to keep only points \( (x, y, \sigma) \) that have a response higher than 50% of the
maximum response across the whole 3D scale space.

(d) [3 points] The radius of a detected blob corresponds to \( \sqrt{2} \) times the detected scale: complete the formula
for the radius \( r \) of each detection, at the end of blob_script.m, in order to plot the detections as circles
of detected radius on top of the image. \( r \) depends on \( \text{sigma}, k \), and the detected scale (use \( \text{smax} \)).

(e) [7 points] Show your detection results for the image sunflowers.jpg and for another image of your
choice containing blobs at various scales. Remark: In case you wonder how to detect white blobs on a
dark background, take the local minima instead of local maxima.