Lens

CCD sensor

3D object

Slide by HyunSoo Park
3D Point Projection (Metric Space)

3D point \((X, Y, Z)\): Focal length in meter

- Pinhole
- Projection plane
- \(f_m\): Focal length in meter

\[
\begin{align*}
\begin{array}{c}
u_{ccd} = f_m \frac{X}{Z} \\
v_{ccd} = f_m \frac{Y}{Z}
\end{array}
\end{align*}
\]

\((X, Y, Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\)

2D projection onto CCD plane
3D Point Projection (Metric Space)

Pinhole = center of projection

$u_{\text{ccd}}, v_{\text{ccd}}$  

$u_{\text{ccd}} = f_m \frac{X}{Z}$  
$v_{\text{ccd}} = f_m \frac{Y}{Z}$

$(X,Y,Z) \rightarrow (u_{\text{ccd}}, v_{\text{ccd}}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$

2D projection onto CCD plane
recall we can paint a picture by

1) placing an image plane in front of us,
2) draw a ray from our eye to a point in the image plane, to form a ray
3) intersect that ray with the 3D world, and mark the intersection point,
4) take the color of that 3D point, and paint it onto the 2D pixel

The ray itself, extended infinitely long, can be also thought of representative of a point in infinity of all 3D lines in direction, and we will see that would allow us to recover the orientation of the camera.
Locating Center of Projection

we will demonstrate how to find COP of your cell phone camera.

1) draw a set of radiating lines on a paper
2) place the camera vertical to the paper
the radiating lines on paper should look like this, in the 3d world
Locating Center of Projection

1) using a live displaying mode, we can see these radiating lines as transformed in to the camera image plane. what does it look like?
2) make sure we keep the camera vertical and move it forward and backward, until the imaged lines look all vertical
3) when we see this we can mark the front of the camera.
Locating Center of Projection

image plane

camera center $C$

lines on ground plane
Locating Center of Projection

-- tracing the radiating lines to their intersection, we obtained a rough estimated of the physical center of projection of your camera

-- we now have an idea of where is the optical center of the camera, how about the focal length itself
Focal Length

Digital SLR cameras have focal length at order of 35mm, or 50 mm. Our cell phone, because it is very thin, has very short focal lengths, on order of a few mm.
Nikon’s 1200-1700mm f/5.6-8P lens.

Professional photographer has lens with much longer focal lengths, on order of 1-2meters.
Zooming, the process of changing focal length, can amplify a scene. We can do the same with a cell phone, just we have to walk a lot to get those images.
Large Focal Length compresses depth

zooming does not only create a sense of magnification, it can also create a sensation of moving, shown here, from 400mm, to 17mm, we can see the size of background objects is much reduced

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**3D Point Projection (Metric Space)**

A 3D point in the world is projected onto a 2D plane using the camera's focal length. The projection equations are:

- \[ u_{ccd} = f_m \frac{X}{Z} \]
- \[ v_{ccd} = f_m \frac{Y}{Z} \]

where:
- \( u_{ccd}, v_{ccd} \) are the 2D coordinates on the CCD plane.
- \( X, Y, Z \) are the 3D coordinates of the point.
- \( f_m \) is the focal length in meters.

The projection plane is parallel to the camera's sensor plane. The center of projection is at distance \( f_m \) from the plane.

The 3D point \((X, Y, Z)\) projects onto the 2D point \((u_{ccd}, v_{ccd})\) on the CCD plane.

2D projection onto CCD plane:

\[ (X, Y, Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \]
3D Point Projection (Metric Space)

- **3D point** $(X,Y,Z)$
- **Projection plane**
- **Projection plane** $(u_{\text{ccd}},v_{\text{ccd}})$
- **Focal length in meter** $f_m$

**Projection Equation**

- $u_{\text{ccd}} = f_m \frac{X}{Z}$
- $v_{\text{ccd}} = f_m \frac{Y}{Z}$

**2D projection onto CCD plane**

- $(u_{\text{img}},v_{\text{img}})$ → $(u_{\text{ccd}},v_{\text{ccd}}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$

**Pixel to Metric Conversion**

- $(u_{\text{img}},v_{\text{img}})$ → $(u_{\text{ccd}},v_{\text{ccd}})$
3D Point Projection (Pixel Space)
3D Point Projection (Pixel Space)
3D Point Projection (Pixel Space)

(u_{ccd}, v_{ccd})

(u_{img}, v_{img})

CCD sensor (mm)

Image (pixel)
<table>
<thead>
<tr>
<th>Imager Sizes</th>
<th>Formats (Type)</th>
<th>~Diag.</th>
<th>Uses</th>
</tr>
</thead>
<tbody>
<tr>
<td>□</td>
<td>1/7&quot; - 1.85 x 1.39mm</td>
<td>2.3</td>
<td>Cell phones, webcams, etc....</td>
</tr>
<tr>
<td>□</td>
<td>1/6&quot; - 2.15 x 1.61mm</td>
<td>2.7</td>
<td>Cell phones, webcams, etc....</td>
</tr>
<tr>
<td>□</td>
<td>1/5&quot; - 2.55 x 1.91mm</td>
<td>3.2</td>
<td>Cell phones, webcams, etc....</td>
</tr>
<tr>
<td>□</td>
<td>1/4&quot; - 3.2 x 2.4mm</td>
<td>4.0</td>
<td>Cell phones, webcams, etc....</td>
</tr>
<tr>
<td>□</td>
<td>1/3.8&quot; - 4.0 x 3.0mm</td>
<td>5.0</td>
<td>P&amp;S DSC</td>
</tr>
<tr>
<td>□</td>
<td>1/3.2&quot; - 4.536 x 3.416mm</td>
<td>5.678</td>
<td>P&amp;S DSC</td>
</tr>
<tr>
<td>□</td>
<td>1/3&quot; - 4.8 x 3.6mm</td>
<td>6.0</td>
<td>Casio QV-8000SX (1.2MP), Epson PhotoPC 700 (1.2MP)</td>
</tr>
<tr>
<td>□</td>
<td>1/2.7&quot; - 5.27 x 3.96mm</td>
<td>6.592</td>
<td>Canon PowerShot A20 (1.92MP), HP PhotoSmart C618 (1.92)</td>
</tr>
<tr>
<td>□</td>
<td>1/2&quot; - 6.4 x 4.8mm</td>
<td>8.0</td>
<td>Olympus C-2100Z (1.92MP), Epson PhotoPC 850Z (1.92)</td>
</tr>
<tr>
<td>□</td>
<td>1/1.8&quot; - 7.176 x 5.319mm</td>
<td>8.932</td>
<td>Nikon Coolpix 995 (3.14MP), Olympus C-4040Z (3.9MP), Canon PowerShot G2 (3.8MP), Sony DSC-S85 (3.8MP)</td>
</tr>
<tr>
<td>□</td>
<td>2/3&quot; - 8.8 x 6.6mm</td>
<td>11.0</td>
<td>Nikon Coolpix 5000 (4.92MP), Sony DSC-F707 (4.92MP), Olympus E-10 (3.7MP), Minolta DIMAGE 7 (4.92MP)</td>
</tr>
<tr>
<td>□</td>
<td>1&quot; - 12.8 x 9.8mm</td>
<td>16.0</td>
<td>Not used in DSCs. Used in some high-end video cameras</td>
</tr>
<tr>
<td>□</td>
<td>Kodak KAF-5100CE CCD</td>
<td>22.28</td>
<td>Olympus announced development of a new camera and new lenses for this 4/3&quot; size. 2814 x 1966 - 6.1MP - 6.8μm pixel</td>
</tr>
<tr>
<td>□</td>
<td>17.8 x 13.4mm (4/3&quot;)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>Foveon X3 F7-35X3-A25B</td>
<td>24.9</td>
<td>Sigma SD9 (X3)</td>
</tr>
<tr>
<td>□</td>
<td>20.7 x 13.8mm</td>
<td></td>
<td>2268 x 1512 = 3.43MP - 9.12μm pixel</td>
</tr>
<tr>
<td>□</td>
<td></td>
<td></td>
<td>1.74x Focal Length Multiplier (35mm film)</td>
</tr>
<tr>
<td>Camera Type</td>
<td>Sensor Size</td>
<td>MP</td>
<td>Pixel Size</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-------------</td>
<td>-----</td>
<td>--------------------</td>
</tr>
<tr>
<td>Canon D30 CMOS</td>
<td>21.8 x 14.5mm</td>
<td>26.2</td>
<td>1.65x Focal Length Multiplier (35mm film)</td>
</tr>
<tr>
<td>Canon D60 CMOS</td>
<td>22.7 x 15.1mm</td>
<td>27.3</td>
<td>1.59x Focal Length Multiplier (35mm film)</td>
</tr>
<tr>
<td>Nikon D100 CCD</td>
<td>23.7 x 15.6mm</td>
<td>28.2</td>
<td>Nikon D100 - 3008 x 2000 = 6.1MP - 7.8μm pixel</td>
</tr>
<tr>
<td>Nikon D1x CCD</td>
<td>25.1 x 16.7mm</td>
<td>30.148</td>
<td>Nikon D1x - 4024 x 1324 = 5.24MP - 5.9 x 11.7μm pixel</td>
</tr>
<tr>
<td>APS Film</td>
<td>27.0 x 17.8mm</td>
<td>32.3</td>
<td>Canon EOS-1D - 2464 x 1648 = 4.06MP - 10.8μm pixel</td>
</tr>
<tr>
<td>Kodak KAF-6303CE CCD</td>
<td>27.8 x 18.5mm</td>
<td>33.4</td>
<td>Kodak 760 - 3088 x 2056 = 6.35MP - 9.0μm pixel</td>
</tr>
<tr>
<td>35mm Film</td>
<td>36.0 x 24.0mm</td>
<td>43.27</td>
<td>35mm film cameras</td>
</tr>
</tbody>
</table>

Canon 1Ds - 4064 x 2704 = 10.99MP - 8.85μm pixel |
Kodak DCS Pro 14n - 4536 x 3024 = 13.7MP - 7.94μm pixel
3D Point Projection (Pixel Space)
3D Point Projection (Pixel Space)

\[(u_{\text{ccd}}, v_{\text{ccd}})\]

\[(u_{\text{img}}, v_{\text{img}})\]

Image (pixel)

Projection of pinhole

CCD sensor (mm)

\[(0,0)\]

\[w_{\text{ccd}}\]

\[h_{\text{ccd}}\]
3D Point Projection (Pixel Space)

$(u_{ccd}, v_{ccd})$  

CCD sensor (mm)

Projection of pinhole

$(u_{img}, v_{img})$  

Image (pixel)
3D Point Projection (Pixel Space)

- (\(u_{\text{ccd}}, v_{\text{ccd}}\))
- (0,0)
- \(w_{\text{ccd}}\)
- \(h_{\text{ccd}}\)
- CCD sensor (mm)
- Projection of pinhole
- (\(u_{\text{img}}, v_{\text{img}}\))
- (0,0)
- \(w_{\text{img}}\)
- \(h_{\text{img}}\)
- Image (pixel)

\((p_x, p_y)\): Image principal point
3D Point Projection (Pixel Space)

Projection of pinhole

\[
\begin{align*}
\frac{u_{\text{ccd}}}{w_{\text{ccd}}} &= \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \\
(0,0) &\quad (u_{\text{img}}, v_{\text{img}})
\end{align*}
\]

(0,0)

(\(p_x, p_y\)): Image principal point
3D Point Projection (Pixel Space)

Projection of pinhole

$$\begin{align*}
    (u_{ccd}, v_{ccd}) &= (0,0) \\
    (u_{img}, v_{img}) &= (p_x, p_y) : \text{Image principal point} \\
    \frac{u_{ccd}}{w_{ccd}} &= \frac{u_{img} - p_x}{w_{img}} \\
    \frac{v_{ccd}}{h_{ccd}} &= \frac{v_{img} - p_y}{h_{img}}
\end{align*}$$
3D Point Projection (Pixel Space)

\[
\begin{align*}
(u_{\text{ccd}}, v_{\text{ccd}}) & \quad \text{CCD sensor (mm)} \\
(u_{\text{img}}, v_{\text{img}}) & \quad \text{Image (pixel)} \\
(0,0) & \quad \text{Projection of pinhole}
\end{align*}
\]

\[
\begin{align*}
\frac{u_{\text{ccd}}}{w_{\text{ccd}}} &= \frac{u_{\text{img}} - p_x}{w_{\text{img}}} & \frac{v_{\text{ccd}}}{h_{\text{ccd}}} &= \frac{v_{\text{img}} - p_y}{h_{\text{img}}} \\
\rightarrow u_{\text{img}} &= u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x & v_{\text{img}} &= v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y
\end{align*}
\]

\((p_x, p_y)\): Image principal point
3D Point Projection (Pixel Space)

\[(u_{\text{ccd}}, v_{\text{ccd}}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\] : Metric projection
3D Point Projection (Pixel Space)

Metric projection

$$(u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y$$
3D Point Projection (Pixel Space)

\[(u_{ccd}, v_{ccd}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) : \text{Metric projection} \]

Pixel projection

\[
\begin{align*}
    u_{img} &= u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img} X}{Z} + p_x \\
    v_{img} &= v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img} Y}{Z} + p_y
\end{align*}
\]
3D Point Projection (Pixel Space)

\[(u_{\text{ccd}}, v_{\text{ccd}}) = (f \frac{X}{Z}, f \frac{Y}{Z})\] : Metric projection

Pixel projection

\[
\begin{align*}
(u_{\text{img}}, v_{\text{img}}) & = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x = f \frac{w_{\text{img}}}{w_{\text{ccd}}} \frac{X}{Z} + p_x \\
v_{\text{img}} & = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z} + p_y
\end{align*}
\]

Focal length in pixel
3D Point Projection (Pixel Space)

Projection plane

O

(X, Y, Z)

(u_{img}, v_{img})

(u_{ccd}, v_{ccd})

(0,0)

CCD sensor (mm)

Image (pixel)

(u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) : Metric projection

Pixel projection

\rightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_x \frac{X}{Z} + p_x

\rightarrow v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_y \frac{Y}{Z} + p_y

Focal length in pixel

where

f_x = f_m \frac{w_{img}}{w_{ccd}}

f_y = f_m \frac{h_{img}}{h_{ccd}}
3D Point Projection (Pixel Space)

Metric projection

\[(u_{\text{cod}}, v_{\text{cod}}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\] : Metric projection

Pixel projection

\[
\begin{align*}
    u_{\text{img}} &= u_{\text{cod}} \frac{w_{\text{img}}}{w_{\text{cod}}} + p_x = f \frac{X}{Z} + p_x \\
    v_{\text{img}} &= v_{\text{cod}} \frac{h_{\text{img}}}{h_{\text{cod}}} + p_y = f \frac{Y}{Z} + p_y
\end{align*}
\]

Focal length in pixel

\[
\begin{align*}
    f_m &= f \frac{w_{\text{img}}}{w_{\text{cod}}} = f \frac{h_{\text{img}}}{h_{\text{cod}}} \\
    w_{\text{img}} &= f_m \frac{w_{\text{img}}}{w_{\text{cod}}} = f_m \frac{h_{\text{img}}}{h_{\text{cod}}}
\end{align*}
\]
3D Point Projection (Pixel Space)

Projection plane

\[(X, Y, Z)\]

\[u_{\text{img}} = f \frac{X}{Z} + p_x\]

\[v_{\text{img}} = f \frac{Y}{Z} + p_y\]
3D Point Projection (Pixel Space)

Projection plane

\[(X,Y,Z)\]

\[(u_{\text{img}}, v_{\text{img}})\]

\[u_{\text{img}} = f \frac{X}{Z} + p_x \quad \rightarrow \quad Z u_{\text{img}} = fX + p_x Z\]

\[v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \rightarrow \quad Z v_{\text{img}} = fY + p_y Z\]
3D Point Projection (Pixel Space)

\[
\begin{align*}
\begin{bmatrix}
Z_{\text{img}} \\
Z_{\text{img}}
\end{bmatrix} &= 
\begin{bmatrix}
f & \frac{p_x}{Z} \\
f & \frac{p_y}{Z}
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
\end{align*}
\]

Where:
- \((u_{\text{img}}, v_{\text{img}})\) are the pixel coordinates in the image.
- \((X, Y, Z)\) are the 3D world coordinates.
- \((u_{\text{ccd}}, v_{\text{ccd}})\) are the pixel coordinates in the CCD sensor.
- \((p_x, p_y)\) are the principal points.

Projection plane:
- \(O\) is the origin.
- \(f\) is the focal length.
- \(Z\) is the depth.

CCD sensor (mm):
- \(w_{\text{ccd}}\) and \(h_{\text{ccd}}\) are the dimensions of the CCD sensor.

Image (pixel):
- \(w_{\text{img}}\) and \(h_{\text{img}}\) are the dimensions of the image.
3D Point Projection (Pixel Space)

\[
\begin{align*}
\text{Pixel space} & \quad \text{Metric space} \\
\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} &= \begin{bmatrix} f & p_x \\ f & p_y \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
u_{\text{img}} &= f \frac{X}{Z} + p_x \\
v_{\text{img}} &= f \frac{Y}{Z} + p_y
\end{align*}
\]

\[
\begin{align*}
Zu_{\text{img}} &= fX + p_x Z \\
Zv_{\text{img}} &= fY + p_y Z
\end{align*}
\]
3D Point Projection (Pixel Space)

Projection plane

\((X,Y,Z)\)

\((u_{\text{img}}, v_{\text{img}})\)

\(u_{\text{img}} = f \frac{X}{Z} + p_x \quad \rightarrow \quad Z u_{\text{img}} = f X + p_x Z\)

\(v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \rightarrow \quad Z v_{\text{img}} = f Y + p_y Z\)

Pixel space

Metric space

\[ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \]

Camera intrinsic parameter:
- metric space to pixel space
3D Point Projection (Pixel Space)

Projection plane

$(X,Y,Z)$

$(u_{\text{img}}, v_{\text{img}})$

$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Z u_{\text{img}} = f X + p_x Z$

$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Z v_{\text{img}} = f Y + p_y Z$

Pixel space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Metric space

Camera intrinsic parameter: metric space to pixel space
Camera Intrinsic Parameter

Metric space

$$[X \ Y \ Z]$$
Camera Intrinsic Parameter

\[
\begin{bmatrix}
  f & p_x \\
  K & p_y \\
  1 & 1
\end{bmatrix}
\begin{bmatrix}
  X \\
  Y \\
  Z
\end{bmatrix}
\]
Camera Intrinsic Parameter

Pixel space

\[ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ p_x \\ p_y \end{bmatrix} \]

Metric space

\[ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \]
2D Inverse Projection

Projection plane

\[
\begin{bmatrix}
u_{\text{img}} \\ v_{\text{img}} \\ 1
\end{bmatrix}
= \begin{bmatrix}
f \\ p_x \\ p_y \\ 1
\end{bmatrix}
\begin{bmatrix}
X \\ Y \\ Z
\end{bmatrix}
\]

Pixel space

Metric space
2D Inverse Projection

2D point == 3D ray

\[
\lambda K^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix}
\]

Pixel space

\[
\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = 
\begin{bmatrix} \lambda \\ \lambda f \\ \lambda \end{bmatrix}
\]

Metric space

\[
\begin{bmatrix} f \\ K \\ p_x \\ p_y \\ 1 \end{bmatrix}
\]

3D ray
2D Inverse Projection

2D point == 3D ray

Pixel space

\[ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_{px} \\ Y \\ 1 \end{bmatrix} \]

\[ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & X \\ 0 & 1 & Y \\ 0 & 0 & Z \end{bmatrix} \]

Metric space
2D Inverse Projection

Pixel space

\[
\lambda \begin{bmatrix}
u_{\text{img}} \\
v_{\text{img}} \\
1
\end{bmatrix} = \begin{bmatrix}
f \\
p_x \\
p_y \\
1
\end{bmatrix} K \begin{bmatrix}X \\
Y \\
Z
\end{bmatrix}
\]

Metric space

\[
\lambda K^{-1} \begin{bmatrix}
u_{\text{img}} \\
v_{\text{img}} \\
1
\end{bmatrix} = \begin{bmatrix}X \\
Y \\
Z
\end{bmatrix}
\]

2D point == 3D ray

The 3D point must lie in the 3D ray passing through the origin and 2D image point.
Exercise

What \( f \) to make the height of Eifel tower appear 960 pixel distance?
Exercise

What f to make the height of Eifel tower appear 960 pixel distance?

$$y_{\text{img}} = f \cdot \frac{Y}{Z} = f_m \cdot \frac{h_{\text{img}}}{h_{\text{ccd}}} \cdot \frac{Y}{Z}$$
Exercise

What f to make the height of Eifel tower appear 960 pixel distance?

\[ y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}} Y}{h_{\text{ccd}} Z} \]

\[ 960 = f_m \frac{1280}{0.0218} \frac{324}{1500} \rightarrow f_m = 0.0757 \text{m} \]
Focal Length

Diagonal viewing angle for 35mm film

Ultra wide-angle | Wide-angle | Standard | Telephoto | Super telephoto

14mm | 20mm | 24mm | 35mm | 50mm | 70mm | 135mm | 200mm | 400mm | 600mm

Normal view seen by the human eye
Exercise

What $Z$ to make the height of Eifel tower appear 960 pixel distance?

$y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$
Exercise

What Z to make the height of Eifel tower appear 960 pixel distance?

\[ y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \]

\[ 960 = 0.05 \frac{1280}{0.0218} \frac{324}{Z} \]

\[ Z = 990.826 \text{ m} \]
What $Z_p$ to make the height of Eifel tower appear twice of the person?

Exercise

$f_m = 50 \text{ mm}$

$Z_p$

$1500$

$1.7 \text{ m}$

$324 \text{ m}$
Exercise

What $Z_p$ to make the height of Eifel tower appear twice of the person?

$$h_e = f \frac{Y}{Z}$$

$$h_p = f \frac{Y_p}{Z_p}$$

s.t. $h_p = \frac{h_e}{2}$
Exercise

What $Z_p$ to make the height of Eifel tower appear twice of the person?

\[
h_e = f \frac{Y}{Z} \quad h_p = f \frac{Y_p}{Z_p} \quad \text{s.t.} \quad h_p = \frac{h_e}{2}
\]

\[
f \frac{Y_p}{Z_p} = f \frac{Y}{2Z} \quad Z_p = 2 \cdot 1500 \cdot \frac{1.7}{324} = 15.74 \text{m}
\]
Where Was I?

Sensor size

\[ y = f \frac{Y}{Z} = f_m \frac{h_{img}}{h_{ood}} \frac{Y}{Z} \]
Where Was I?

\[
y = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}
\]
Where Was I?

Circa 1984
Where Was I?

Circa 1984
Where Was I?

Circa 1984

$670 \text{ pix}$

$250 \text{ pix}$

$f_m = 50 \text{ mm}$

$0.9 \text{ m}$

$324 \text{ m}$
Where Was I?

Circa 1984

\[ y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}} Y_1}{h_{\text{obj}} Z_1} \]
Where Was I?

Circa 1984

\[ y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}}}{h_{\text{obj}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{obj}}} Y_1 \]
**Where Was I?**

Circa 1984

---

The Eiffel Tower:
- Height: 324 m
- Width: 1280 m

The boy:
- Height: 250 pix
- Width: 670 pix

- **Formula:**
  
  \[
  y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}} Y_1}{h_{\text{cod}} Z_1}
  \]

  Where:
  - \( f_m = 50 \text{ mm} \)
  - \( h_{\text{img}} = 0.05 \text{ m} \)
  - \( h_{\text{cod}} = 0.0218 \text{ m} \)
  - \( Y_1 = 1 \text{ cm} \)
  - \( Z_1 = 8.03 \text{ m} \)

  Thus:

  \[
  y_1 = f_m \frac{h_{\text{img}} Y_1}{h_{\text{cod}} Y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{ m}
  \]
Where Was I?

Circa 1984

\[ y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}} Y_1}{h_{\text{cd}} Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}} Y_1}{h_{\text{cd}} y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{m} \]

\[ y_2 = f \frac{Y_2}{Z_2} = f_m \frac{h_{\text{img}} Y_2}{h_{\text{cd}} Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}} Y_2}{h_{\text{cd}} y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079 \text{m} \]
Where Was I?

\[ y_2 = f \frac{Y_2}{Z_2} = f_m \frac{h_m}{h_{\text{mod}}} \frac{Z_2}{Z_2} = f_m \frac{h_m}{h_{\text{mod}}} \frac{Y_2}{Y_2} = 0.05 \frac{1280}{0.0218 \cdot 670} = 1079 \text{m} \]
Where Was I?
Focal Length

$f$  $d$

Strong perspective
Focal Length

\[ f \quad d \]

Weak perspective
Focal Length

\[ f \quad d \]
Focal Length
Angle of View

7.5 mm
15 mm
17 mm
20 mm
24 mm
28 mm
35 mm
50 mm
85 mm
100 mm
135 mm
200 mm
300 mm
400 mm
500 mm
600 mm
800 mm
Dolly Zoom (Vertigo Effect)

VERTIGO (1958)
Dolly Zoom (Vertigo Effect)

JAWS (1975)
Dolly Zoom

Given focal length \( f_m = 100 \text{mm} \), what \( Z_{100} \) to make the height of the person remain the same as \( f_m = 50 \text{mm} \)?

\[ Z_{50} = 157.41 \text{m} \]
Dolly Zoom

Given focal length ($f_m=100\text{mm}$), what $Z_{100}$ to make the height of the person remain the same as $f_m=50\text{mm}$?

\[
h_{50} = f_{50} \frac{Y}{Z_{50}} \quad h_{100} = f_{100} \frac{Y}{Z_{100}} \quad \text{s.t.} \quad h_{100} = h_{50}
\]
Given focal length \( f_m = 100\text{mm} \), what \( Z_{100} \) to make the height of the person remain the same as \( f_m = 50\text{mm} \)?

\[
\begin{align*}
h_{50} &= f_{50} \frac{Y}{Z_{50}} \\
h_{100} &= f_{100} \frac{Y}{Z_{100}} \\
s.t. \quad h_{100} &= h_{50}
\end{align*}
\]

\[
\begin{align*}
Z_{100} &= \frac{f_{100}}{f_{50}} Z_{50} \\
Z_{100} &= \frac{100}{50} 157.41 = 314.8\text{m}
\end{align*}
\]
Where am I with Dolly Zoom?

\[ h_1 = 400\text{pix} \]

\[ h_2 = 120\text{pix} \]
Where am I with Dolly Zoom?

How far I need to step back with zoom factor x2?

How will \( h_2 \) change?
Where am I with Dolly Zoom?

How far I need to step back with zoom factor x2?
How will $h_2$ change?

\[ h_1 = 400 \text{pix} \quad h_2 = 120 \text{pix} \]

\[ H_1 = 4 \text{m} \quad H_2 = 6 \text{m} \]

\[ h_1 = \frac{h}{z} \quad h_2 = \frac{h}{z} \times 2 \]

\[ h_1 = \frac{400}{z} \]

\[ h_2 = \frac{120}{z} \times 2 \]

\[ \frac{4}{z} = \frac{120}{z} \times 2 \]

\[ z = \frac{4}{240} \]

\[ z = \frac{1}{60} \text{m} \]

\[ z = \frac{1}{60} \times 100 \]

\[ z = \frac{100}{60} \]

\[ z = \frac{5}{3} \text{m} \]

\[ z = \frac{5}{3} \times 100 \]

\[ z = \frac{500}{3} \]

\[ z = 166.67 \text{ m} \]
Where am I with Dolly Zoom?

Unkowns: $f$, $d_1$

How far I need to step back with zoom factor $x2$?
How will $h_2$ change?
Where am I with Dolly Zoom?

Unknows: $f$, $d_1$, $\Delta d$

How far I need to step back with zoom factor $x2$?
How will $h_2$ change?
Where am I with Dolly Zoom?

Equations:

\[ h_1 = f \frac{H_1}{d_1} \]

How far I need to step back with zoom factor x2?
How will \( h_2 \) change?
Where am I with Dolly Zoom?

Equations:
\[ h_1 = \frac{f H_1}{d_1} \]
\[ h_t = 2f \frac{H_1}{\Delta d + d_t} \]

Unknowns: \( f, d_1, \Delta d \)

How far I need to step back with zoom factor \( x2 \)?
How will \( h_2 \) change?
Where am I with Dolly Zoom?

Ununknowns: $f$, $d_1$, $\Delta d$

Equations:

\[
h_1 = f \frac{H_1}{d_1} \quad \text{and} \quad h_1 = 2f \frac{H_1}{\Delta d + d_1} \quad \rightarrow \quad \Delta d = d_1
\]

$H_1 = 4m$  
$H_2 = 6m$  
$d = 2m$  

How far I need to step back with zoom factor x2?  
How will $h_2$ change?

Top view

Image plane

Camera

$d_1$

$\Delta d$

$2f$

$h_1 = 400\text{pix}$  
$h_2 = 120\text{pix}$

$\Delta d = d_1$
Where am I with Dolly Zoom?

Equations:
\[ h_1 = f \frac{H_1}{d_1} \]
\[ h_2 = f \frac{H_2}{d_1 + d} \]
\[ h_2 = 120\text{pix} \]
\[ H_1 = 4\text{m} \]
\[ H_2 = 6\text{m} \]
\[ d = 2\text{m} \]
\[ d_1 \]
\[ \Delta d \]

Unknowns: \( f \), \( d_1 \), \( \Delta d \)

How far I need to step back with zoom factor x2?
How will \( h_2 \) change?
Equations:

\[ h_1 = f \frac{H_1}{d_1} \]
\[ h_2 = f \frac{H_2}{d_1 + d} \]
\[ d_1 = \frac{1}{1 - \frac{h_2 H_2}{h_1 H_1}} \]
\[ d = 2 \text{m} \]
\[ \Delta d = 2.5 \text{m} \]

Unknowns: \( f, d_1, \Delta d \)
Equations:

\[ h_1 = f \frac{H_1}{d_1} \]

\[ h_2 = f \frac{H_2}{d_1 + d} \]

\[ d_1 = \frac{1}{h_2 \frac{H_1}{h_1 H_2}} \]

\[ \Delta d = 2.5 m \]

Unknowns: \( f, d_1, \Delta d \)
Equations:

\[ h_1 = f \frac{H_1}{d_1} \]

\[ h_2 = f \frac{H_2}{d_1 + d} \]

\[ h_1' = 2f \frac{H_1}{\Delta d + d_1} \]

\[ h_2' = 2f \frac{H_2}{\Delta d + d_1 + d} \]

Unknowns: \( f, d_1, \Delta d \)

How far I need to step back with zoom factor x2?

How will \( h_2 \) change?
How far I need to step back with zoom factor x2?

How will $h_2$ change?

Equations:

$$h_1 = f \frac{H_1}{d_1}$$

$$h_1 = 2f \frac{H_1}{\Delta d + d_1}$$

$$h_2 = f \frac{H_2}{d_1 + d}$$

$$h_2' = 2f \frac{H_2}{\Delta d + d_1 + d} = 429 \text{pix}$$

$\Delta d = 2.5 \text{m}$ $f = 250 \text{pix}$

Unknowns: $f$, $d_1$, $\Delta d$