Strategic Network Formation Experiment
Networked Life (NETS 112)
Fall 2015
Prof. Michael Kearns

Experiment to be held in class, Thursday October 29
Follow-up survey due via email response Saturday, October 31

While you are encouraged to think carefully about the strategies you will adopt before the experiment, you are NOT permitted to communicate with your classmates in any way prior to the experiment. Doing so will be treated as an instance of cheating.

This document describes a communal class experiment that will count for course credit. Your score on the assignment will be exactly your expected payoff in the game described below. If you fail to participate, your score will be 0. Following the experiment, you are also required to complete an email survey as part of the assignment. Please read this document carefully and in its entirety.

The Scenario

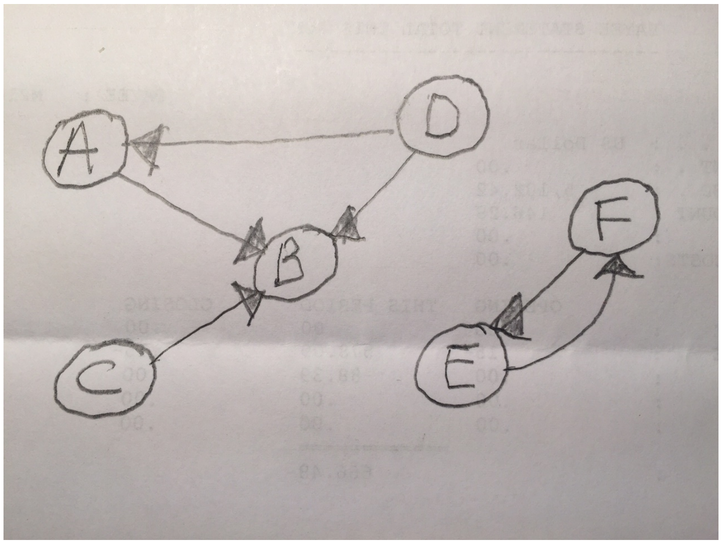
For this assignment you should consider yourself a player in a game played with all the other members of the class. Your high-level goal is to form edges or links to other players in order to able to "reach" as many other players as possible. By "reach" we mean that another player is in the same *connected component* as you. You are free to form edges to as many other players as you like --- but you have to pay for them. So the first tension in the game is that by purchasing edges to others, you're more certain they will be in your connected component; but the more edges you buy, the more you pay.

Let's first formalize this basic version of the game (which is NOT the one you will play; there is an important twist below). The decision you need to make is very simple: you have to specify exactly which other players in the class you are going to purchase edges to. Once everyone has specified their individual edge purchases, we look at the graph G that is formed by the *collective* edge purchases. The payoff of a specific player P is then defined to be:

(size of P's connected component) – α x (number of edges purchased by P)

where α is the cost per edge.

Let's work through a simple example, in which $\alpha = 1$ and there are 6 players: A(lice), B(ob), C(huck), D(avid), E(va) and F(iona). Suppose that the edge purchasing decisions by these players forms the collective network shown in the image below, where an arrow from player/vertex X to player/vertex Y means that X purchased an edge to Y. Then the payoffs to the players would be as follows:



A(lice): Her connected component is of size 4 (a player is included in their own connected component), and she purchased 1 edge (to B(ob)), so her payoff is 4 – 1 = 3.

B(ob): His connected component is of size 4, and he purchased no edges, so his payoff is 4 – 0 = 4.

C(huck): 4 – 1 = 3.

D(avid): 4 – 2 = 2.

E(va): 2 – 1 = 1.

F(iona): 2 – 1 = 1.

Some important points about this example and the game more generally. First of all, edge purchasing is *unilateral* --- i.e. you don't need permission or reciprocation from another player to purchase an edge to them --- but the benefit of an edge purchase is *bilateral*, meaning that both players are connected to each other through an edge purchase by either of them. Thus although we have drawn the edges as arrows above to indicate who purchased each edge, we compute connected components on the *undirected* graph of edge purchases.

Note that in the example above E and F both purchased edges to each other. If either one of them (but not both) dropped the edge to the other, their payoff would increase --- they would remain connected, but would save on the edge purchase. Similarly but a bit more subtly, note that any one of the edge purchases A→B, D→A, D→B could be dropped and save the buyer that cost without changing the size of their connected component. But dropping any two of these edges would break up the component {A,B,C,D}.

It is important to realize that you may not need to purchase edges to remote players in order to enjoy connectivity benefits to them. For example, C purchases only the edge to B, but A and D are also included in C's payoff due to the edge purchases by others. B didn't buy a single edge, and has the highest payoff of all in this example.

The Twist: Part One

If only things were as simple as the game described above… unfortunately, we live in a dangerous world, with adversaries always trying to attack and destroy the networks we seek to form.

To model this grim reality, we introduce a powerful Adversary who has the luxury of launching his attack *after* the players have formed their network. Furthermore the Adversary also gets to see the entire network. The Adversary's attack acts like a fatal infection that destroys every player it can reach through the network. More precisely, the Adversary is allowed to pick a single "seed" player/vertex to infect. This infection then spreads throughout the entire connected component of the seed, killing all of them. The Adversary, being rational and desiring to inflict as much carnage as possible, *always chooses to attack a vertex in the largest connected component* of the graph formed by the players.

Thus, in our example above, the Adversary would attack some player in the component {A,B,C,D} since that is the largest component. It doesn't matter which specific player in that component is attacked, since they will all die anyway.

How does the presence of the Adversary change the payoffs to players? Simple: if a player is killed by the Adversary, their connectivity benefit is zero (but they still pay for any edges they bought); if they are not killed by the Adversary, their payoff is unchanged. Let's go through our example above again, but now with the Adversary. The payoffs would now become:

A: She is in the largest component, so she dies and gets 0 connectivity benefit. But she did purchase one edge. So her payoff would be 0 – 1 = -1.

B: Also dies, bought no edges. Payoff is 0 – 0 = 0.

C: Dies, bought one edge, payoff is 0 – 1 = -1.

D: Dies, bought two edges, payoff is 0 – 2 = -2.

E: Lives, so payoff remains 2 – 1 = 1.
F: Lives, payoff remains 2 – 1 = 1.

So the introduction of the Adversary introduces another tension: while you'd like to be able to reach lots of other players, if you reach "too many" by being in the largest component, you die and have zero or even negative payoff.

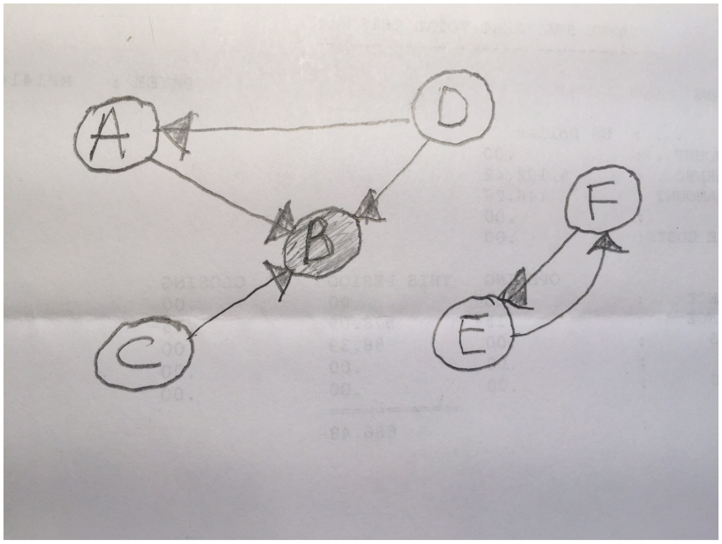
The Twist: Part Two

The Adversary is indeed powerful and fearsome, but mercifully there is a ray of hope for our players. It turns out there is way of protecting or *immunizing* yourself against the Adversary's attack. In our capitalist society/game, this of course also must be paid for. If a player purchases immunization, they are protected from the Adversary's attack --- not only will the infection not kill them, it cannot pass through them either. Thus player P immunizing may also have the effect of protecting other players who do not immunize, if the only way the infection could reach them is through P.

A player's decision or action now consists of two parts: which other players to purchase edges to, and whether or not to purchase immunization.

Of course, in the same way that our omniscient Adversary saw the entire network before deciding where to attack, he also gets to see everyone's immunization decisions as well. How will the rational, carnage-loving Adversary behave now? It's simple: he will first remove or delete all players/vertices who have immunized --- they cannot be killed, and the Adversary's attack cannot pass through them. The Adversary will then look at the component structure of the graph after removal of the immunes, and once again attack and destroy the largest connected component.

Let's revisit our example with this immunization twist. Let's assume that only one player --- say B(ob) --- chooses to purchase immunization, represented by the shading on vertex B in the revised figure below:



What will the Adversary do for this network? If we remove B and his edges from this graph, the connected components become {A,D}, {C} and {E,F}. In this case, there is thus a tie for the largest connected component. The Adversary is indifferent between them, so will attack each of them with probability ½. *More generally, if there are K components that are all tied for the largest, the Adversary will destroy each with probability 1/K.*

What about the payoffs to the players? The final payoff of a player P is defined as follows:

(***expected*** size of P's connected component)

– α x (number of edges purchased by P)

– β (if and only if P purchased immunization)

Here by expected size of P's component, we mean the expectation over the Adversary's random selection of the largest component to attack in case of ties.

Let's revisit our running example one more time, again assuming edge cost α = 1 and immunization cost β = 2. Remember that after removal of the immunized B, the Adversary attacks each of {A,D} and {E,F} with probability ½. The payoffs would thus be as follows:

A: She purchased one edge and no immunization. With probability ½ she dies and gets connectivity benefit of 0, and with probability ½ she lives and gets connectivity benefit of 4 (remember that the removal of the immunized B was just a thought experiment by the Adversary to decide where to attack). So her payoff is then ½(0) + ½(4) – 1 (edge purchase) – 0 (no immunization) = 2 – 1 = 1.

B: Purchased no edges but bought immunization. With probability ½ the component {A,D} is killed and B is connected only to C; with probability ½ the component {A,B,C,D} is unperturbed. So B's payoff is ½(2) + ½(4) – 0 (no edges bought) – 2 (immunization) = 1 + 2 – 2 = 1.

C: ½(4) + ½(2) – 1 – 0 = 2 + 1 – 1 = 2.

D: Similar to A, but bought an additional edge: ½(0) + ½(4) – 2 – 0 = 2 – 2 = 0.

E and F: With probability ½ they both die, and with probability ½ they both live. Their payoffs are thus ½(0) + ½(2) – 1 – 0 = 1 – 1 = 0.

So immunization introduces yet a third tension: you can protect yourself, but it costs; and your immunization may protect or help others who don't pay for immunization. Conversely, even if you immunize you can lose connectivity benefits due to others not immunizing. For instance, in our example the immunizing B loses connectivity to A and D with probability ½.

Details, Logistics and Ground Rules for the Experiment

In class on Thursday, October 29, we will conduct an interactive version of the game described above. Your score for the experiment will count for course credit, and will be exactly your expected payoff as described above, under the following choices:

α = 5 points

β = 20 points

In other words, each player you remain connected to following the Adversary's attack (which again may be randomized) is worth 1 point; each edge you purchase deducts 5 points from your score; and purchasing immunization deducts 20 points from your score. Scores for the experiment will be determined by taking everyone's edge purchases and immunization decisions and running an algorithm for computing expected payoffs.

At the start of the experiment, you will be given a piece of paper with:

- A place to clearly write your name
- An area to write down the names of the other players you decide to purchase edges to
- A place to indicate whether you choose to purchase immunization

You will be given the entire class period to reach decisions about your edge purchases and immunization. During this time, you will be allowed to freely interact and have any discussions you like with any of your classmates, with the ground rule that *all conversation must be quiet and subdued --- no shouting or loud conversation or comments will be permitted,* and violations will result in a 0 on the assignment. Note that while you are free to interact with anyone and any way you like, at the end of the session you will turn in only the sheet of paper with your decisions on it. In particular, there is no mechanism provided for entering any binding agreements with your classmates.

Email Survey

Please try to carefully remember the nature of the conversations you have with your classmates, how many you speak to, and your thought processes and strategy during the experiment. Please also try to observe and remember the behavior and apparent strategies of your classmates. Following the experiment, you will receive an email survey from Prof. Kearns asking you to reflect on these observations. *In order to receive credit for the experimental assignment, you must return the email survey by midnight on Saturday, October 31.*